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THE INFLUENCE OF SUN SPOTS ON RADIO RECEPTION.*

BY

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Of all astronomical bodies the sun is by far of the greatest immediate concern to human beings. Literally in him we live, and move, and have our being. Every square yard of the earth's surface exposed to direct sunshine receives energy at the rate of more than one and a half horse-power; the whole earth receives from the sun heat at such a rate that if converted into doing work it would represent the equivalent of 230,000,000,000,000 horse-power. With the exception of a few experimental solar engines once set to work for irrigation purposes in arid regions, man has practically made no attempt to convert, directly, solar rays into mechanical effort. Perhaps when oil is running low and coal is $100 a ton we may learn to tap profitably this abundant source.

When we reflect, further, that, after all, the whole earth can intercept less than one-billionth of the sun's total output, we realize our complete inability to conceive of this stupendous and seemingly inexhaustible supply. To determine conditions at the surface of the sun and to measure its temperature, which is about 12,000° F., is a far simpler task than to divine the source of its enormous energy, or to conjecture just what is going on in the interior.

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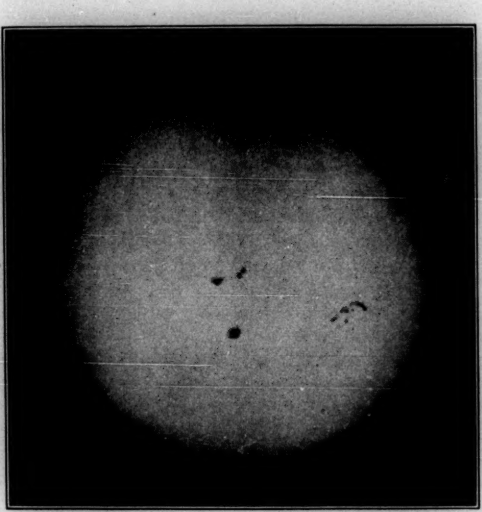
* Presented at a meeting held Thursday, February 13, 1930.

However, the sun is a typical star, and, thanks to the researches of Eddington at Cambridge University, we are no longer as ignorant as we once were concerning the interior of the stars, and our best guess as to the source of their radiant energy leads us into the very structure of the matter of which stars are made. We may picture the interior of the sun as a veritable hurly-burly of atoms and electrons, flying hither and thither at terrific velocities which rapidly increase as the sun's center is approached, where the temperature is of the order of eighty million degrees. From this hot interior of the sun must ultimately arise the source of solar radiation, which not only manifests itself in familiar light and heat, but in a wide variety of electro-magnetic disturbances, whose far-reaching effects we are just beginning to appreciate.

FIG. 1.

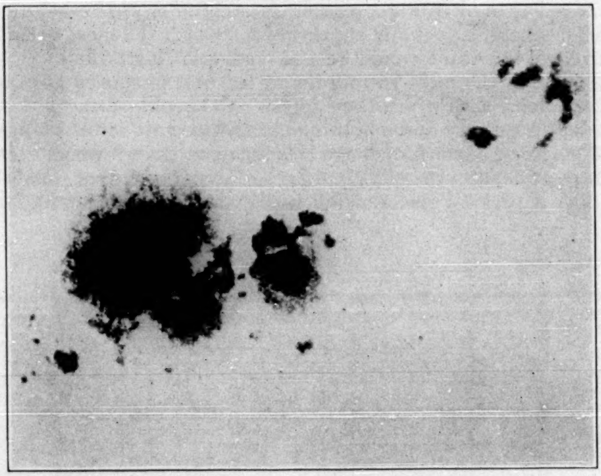


The sun photographed at Perkins Observatory

Some radio enthusiasts who have been long at the game may sense that of late years, long distance reception has not been coming in as it did in the early days of broadcasting, five or six years ago. This is the more significant when we consider that the output of the broadcasting stations has been increased immensely and that great improvement has been made in receiving-sets over this interval.

Studies during the last few years indicate that there are cosmic causes at work which may profoundly influence the electrical state of our atmosphere which these radio waves traverse. Probably the sun is the one astronomical body most responsible for changes in our terrestrial affairs. Every radio fan knows that day-time reception is vastly poorer than night-time reception in the broadcasting zone. Here is the most obvious exhibition of the effect of the sun's rays upon radio. On the other hand, both day and night reception vary greatly from time to time for what has often seemed no good

FIG. 2.



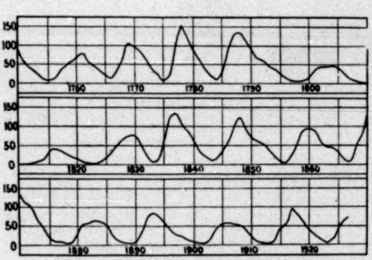
An enlarged view of a typical group of sunspots.

reason at all. It is from relatively very recent researches that we have come to believe much of the cause for this varying degree of reception is to be found in the sun's atmosphere itself.

When we examine the sun's surface through the telescope, we find that it presents a strange, mottled, or granular appearance. In this mottled surface there develop now and then dark patches, often growing into huge black areas surrounded by a somewhat shaded region called the penumbra. These dark areas are the sunspots. Whatever may be ultimately accepted as the best explanation of the spots, one cannot go far wrong in picturing a sunspot as a terrific storm in the sun's atmosphere, a cyclonic whirlwind for which the most violent tropical hurricane would be a microscopic illustration.

One of the most extraordinary features of sunspots is the periodicity with which they appear on the solar surface. For nearly a century and a half sufficiently accurate records of the appearance of sunspots have been made so that, if we plot the degree of spottedness of the solar surface year by year, we discover a periodic rise and fall in the stormy condition of the sun's surface spanning approximately eleven years. We are now not far from what we call a sunspot maximum. About six years ago sunspots were very scarce, and when they occasionally appeared, were very small and insignificant affairs.

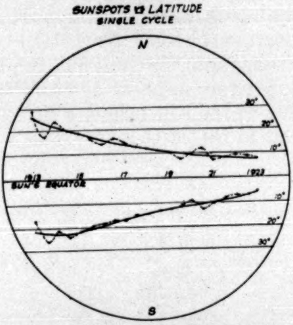
FIG. 3.



The sun-spot curve, according to Wolf's sun-spot numbers. (From data by Wolfer.)

Curiously enough, at the beginning of a sunspot cycle the spots appear on the sun's surface at relatively high latitudes

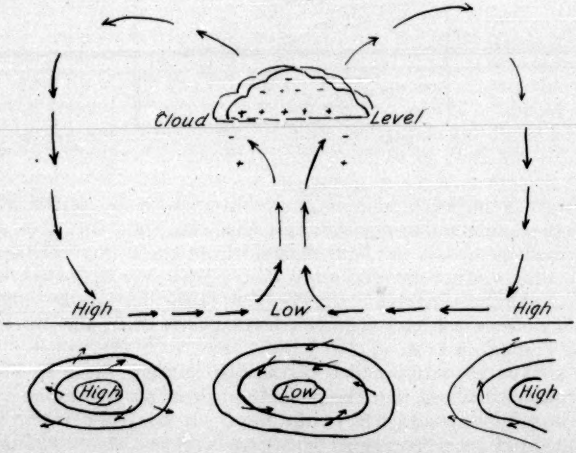
FIG. 4.



As the eleven-year sunspot cycle waxes the latitude of the spots decreases.

and as the cycle progresses they increase in size and number and break out at successively lower latitudes on the solar sphere, a given cycle of spots finally disappearing just a few degrees from the solar equator.

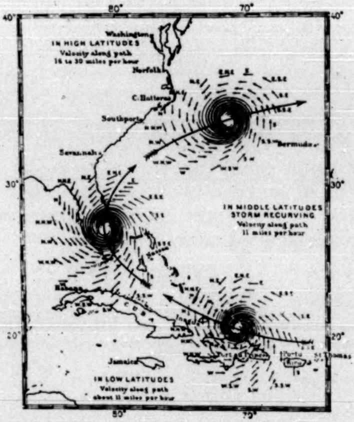
FIG. 5.



Formation of a thunderstorm is analogous to the formation of a sunspot. Note the vertical whirls about "High" and "Low" pressure areas.

The true character of sunspots as magnetic whirls in the solar atmosphere was first established by Hale of the Mount Wilson Observatory, in 1908. By means of the newly invented spectro-heliograph, Hale was able to photograph different layers in the solar atmosphere and establish the existence of vortices similar to the whirlwinds which are characteristic of cyclonic storms in the earth's atmosphere. Furthermore, by analyzing with polarizing apparatus the character of the rays of light radiating from the sunspots, Hale was able to demonstrate that the light emitted from the center of these gigantic whirls betrayed unmistakably that they were electromagnetic poles and that the doubling of the lines in the spectrum over sunspots was due to the magnetic effect announced by Zeeman in 1896.

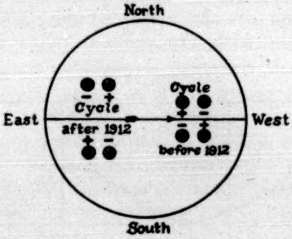
FIG. 6.


80°
70°
40°
40°
30°
30°
20°
20°
80°
70°
Washington
IN HIGH LATITUDES
Velocity along path
16 to 30 miles per hour
Norfolk
C. Hatteras
Southport
Savannah
Bermuda
IN MIDDLE LATITUDES
STORM RECURVING
Velocity along path
11 miles per hour
Habana
CUBA
Jamaica
Port au Prince
Porto Rico
St. Thomas
IN LOW LATITUDES
Velocity along path
about 11 miles per hour

Tropical storms on the earth frequent latitudes corresponding to the latitudes of many major spots on the sun.

With the reappearance of the last sunspot cycle it was firmly established that the polarity of these spots completely reverses from one cycle to the next. It has been recently suggested by a Norwegian scientist, Bjerknes, that the sunspots are the visible ends of a tubular vortex which may extend east and west for great distances below the sun's surface. A reversal in the direction of whirl in this supposed vortex would account for a reversal of the magnetic polarity of the sunspots with the change of cycle.

FIG. 7.



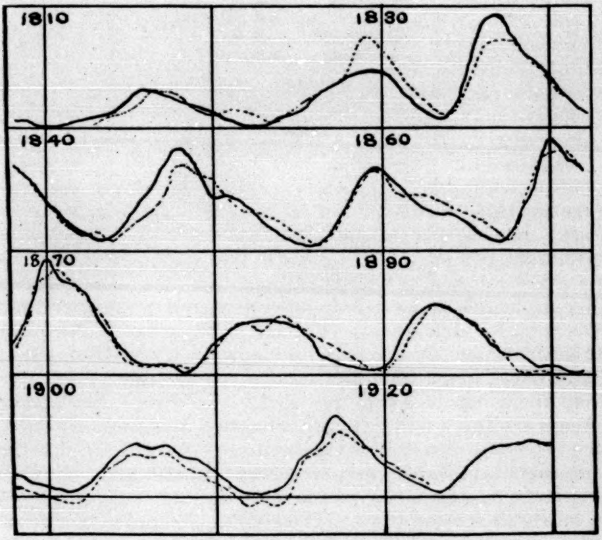
Showing reversal in the magnetic polarity of the spots with change in cycle.

No completely satisfactory explanation of the ultimate origin of these whirls has yet been made. There is one peculiarity, however, in the sun's behavior which doubtless has an important bearing on this point. While the sun rotates on its axis from west to east in common with the axial rotation of other bodies in the solar system, its period of rotation is not the same for different parts of the solar surface. Near the equator the sun rotates once on its axis in a period of almost twenty-four and a half days, whereas in latitude 35° the motion of the spots across the surface indicates that almost twenty-six and a half days are consumed in a single rotation. Spectroscopic observations make it possible to determine the rate of rotation in regions of higher latitudes than those in which the spots appear. In latitude 60° the rotation period is nearly thirty-one days. The continual slipping of the atmospheric layers of lower latitude past those of higher latitude must result in the formation of eddy currents favorable to the formation of cyclonic whirls, thus producing sunspots.

The mention of sunspots invariably raises the question of a possible connection between the spots on the sun and terrestrial phenomena. Some statisticians with an insatiable appetite for correlations have attempted to connect with sunspots almost every cycle in world affairs from fluctuations in the New York stock market to the fecundity of rabbits in Northern Canada. In the popular mind almost every world catastrophe has sooner or later been attributed to sunspots, from a Florida hurricane to the great World War, both of which, by the way, did culminate around a sunspot maximum.

But seriously, there are to the scientist certain well recognized phenomena on the earth which pass through cycles whose correlation with the sunspot cycle is unmistakable.

FIG. 8.



Graph showing correlation of sunspot numbers to magnetic effects on the earth.

For more than a century and a half records of the numbers of sunspots have been kept and afford data for a study of their periodicity over a range of about fifteen eleven-year cycles. For more than a century records of the variation in the earth's magnetism have been made and preserved. The remarkable correlation of sunspots with magnetic changes on the earth is at once apparent when we make a graph of the number of sunspots and compare this with a similar graph for changes of the compass needle (Fig. 8). Simultaneously with the so-called magnetic storms, which are wont to sweep the earth upon the appearance of great sunspot activity, we witness frequent and brilliant displays of the aurora borealis.

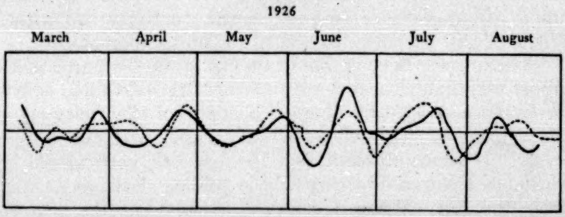
The auroral light is due to an electronic discharge in the upper and highly rarified atmosphere of the earth and is most probably activated by charged particles of electricity emanating from the sun, whose activity varies with the sunspot cycle. It seems probable that the magnetic vortical whirl of sunspots acts as a directing field in guiding electrons escaping from the sun. When a conspicuous spot appears near the center of the solar disk, and is therefore approximately in line with the earth and the sun's center, there is a particularly good chance of the ejected electrons striking the earth's atmosphere and causing an ionization, or electrification, of the upper atmosphere giving rise to an auroral display. At the same time the induced earth currents will distort the earth's magnetic field, causing the small variations in the compass needle so characteristic of a "magnetic storm."

While for many generations scientists have recognized the recurrent cycle in solar activity and the magnetic changes in the earth, never before the present period of sunspot activity has it been possible to study so thoroughly the changing degree of electrification in the earth's atmosphere with the coming and going of the spots across the solar disk. All this has come about by the development of the radio.

The same electric disturbances which alter the earth's magnetic field and produce the displays of the auroræ, or northern lights, so change the electrical state of our atmosphere that the radio waves are also affected to a very marked degree by the coming and going of the gigantic solar cyclones.

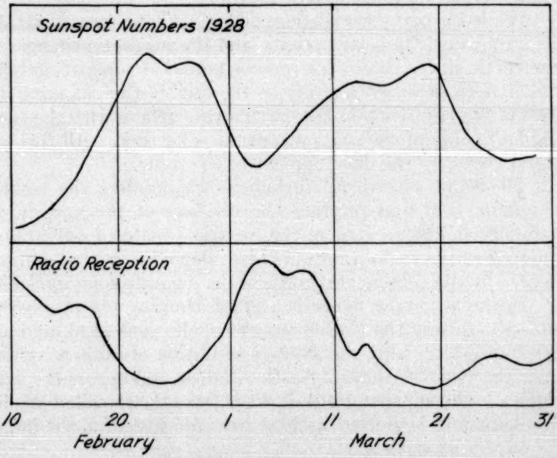
In the adjoining figure is a graph showing the number of sunspots during the twelve months of the year 1926, and another graph showing the average condition of radio reception over the North Atlantic, South Atlantic and across the continent. The sunspot graph is made from the so-called Wolfer numbers, and is plotted with an inverted scale, i.e. the larger the numbers the shorter the ordinate of the curve. These Wolfer numbers are based upon the number of spots visible on the sun's surface at a given time and to some extent upon their area, but do not take into account the position of the spot on the sun's disk. The general run of these graphs indicates that radio reception is distinctly impaired by an increase in the sunspot numbers.

FIG. 9.



Curve showing correlation of sunspots with radio reception: Dotted curve, the inverse of sunspot numbers; full curve, relative intensity of radio reception on transatlantic, South American, and continental reception.

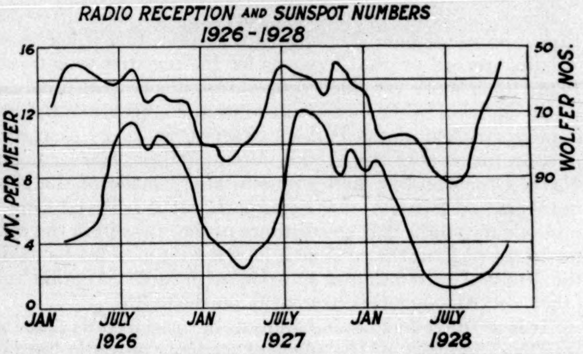
FIG. 10.



Showing that the intensity of radio signals varies inversely with numbers of sunspots. Based on data received in Cambridge from WBBM Chicago.

Quantitative measurements of radio reception since 1926 seem to have established beyond much doubt that long distance night reception in the broadcast zone is in general poor when sunspots are numerous, and good when the spots are few (Fig. 10). The quantitative measurement of radio reception in the broadcast zone, was begun by Mr. G. W. Pickard in his private laboratory in Newton Centre, Massachusetts, in February, 1926. Great credit is due to Mr. Pickard for his contribution in this field and his stimulus to other workers. In February, 1928, a duplicate set of apparatus was installed at the Astronomical Laboratory at Harvard University, and the measurements carried on there under the direction of the author. Simultaneous records made for a short time at both receiving stations gave the necessary reduction factor for rendering those two series of observations comparable. The investigations at the Newton Centre laboratory were then shifted from the broadcast zone to the region of eighteen kilocycles.

FIG. 11.



Upper curve is inverse of running mean of sunspot numbers. Lower curve running mean of radio signal strength received at Boston from WBBM Chicago.

The adjacent figure shows in the upper graph the inverted curve of sunspot numbers, and in the lower graph, the intensity of the carrier wave of WBBM broadcasting station, as received in the vicinity of Boston for 1926–1929, and is based upon the results of measurements made by Mr. G. W. Pickard and the author, working in Newton Centre and in Cambridge, Massachusetts. The radio intensities are recorded in terms of microvolts in the antenna of the receiving circuit.

In making plans for research at the new Perkins Observatory at Delaware, Ohio, it was decided to use the opportunity to further the present investigation by establishing an additional station in the middle west, at one-third the distance from WBBM over which we were operating in Massachusetts. Another observer, Mr. Brown of Pasadena, California, is gathering similar data from a Pacific coast station.

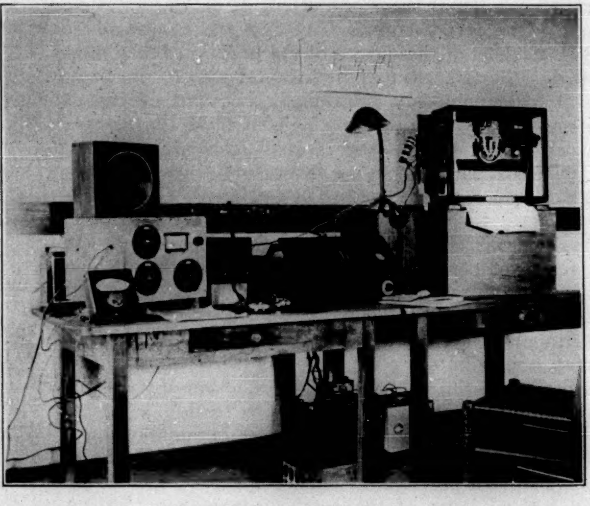
Now, every night, Sundays and holidays included, stations in Massachusetts, in Ohio, and in California, tune in on a prescribed wave-length to study the effect of the day's solar radiation upon the electrical state of the earth's atmosphere. Not trusting to any personal impressions as to whether reception is excellent, good, fair, or poor, an attendant closes the key to the automatic recorder, whose pen, with an impersonal but almost uncanny intelligence writes an unprejudiced record of the intensity of the incoming waves. It is with utter disregard for astronomical or electrical theories that it leaves its indelible record of what happens for the scientist to analyze. Plans for a more extended coöperation are now in progress.

In addition to the measurement of radio reception, the sun is photographed at the Perkins Observatory every clear day in coöperation with the Yerkes, Mount Wilson, Harvard, and Naval Observatories, and a careful study made of the size, numbers and location of the sunspots. It is believed from a preliminary study that the distance of the spots from the center of the disk, or the sun-earth line, is an important factor in the study of correlation of sunspots with radio reception and other electro-magnetic phenomena on the earth.

It is in the reduction and analysis of these records that the startling discovery of the effect of sunspots on radio has become established.

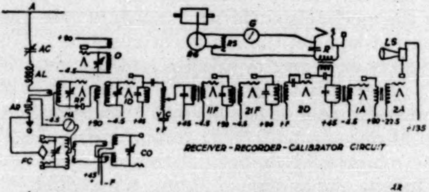
The radio apparatus in use at the Perkins Observatory is a super-heterodyne receiver especially constructed for the purpose, and feeding into a self-recording galvanometer which registers the strength of the carrier wave received from the

FIG. 12.

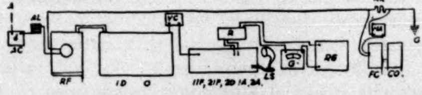


Automatic recording radio apparatus at Perkins Observatory.

FIG. 13.


A
AC
AL
AR
+90
-4.5
O
RF
ID
MA
VC
+F
IIF
21F
2D
1A
2A
+45
+135
-22.5
G
RS
RG
R
LS
FC
CO
-F

RECEIVER - RECORDER - CALIBRATOR CIRCUIT


A
AL
AC
RF
ID O
VC
R
IIF, 21F, 2D 1A, 2A.
LS
G
RG
AR
MA
FC
CO.
G

ARRANGEMENT OF APPARATUS

Diagram of radio receiving circuit for recording intensities in the carrier wave.

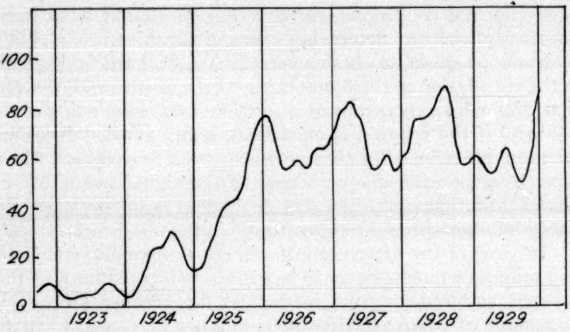
broadcasting station of WBBM, Chicago. The apparatus is so designed that the modulations of the carrier wave do not affect the record appreciably and the results obtained are independent of the nature of the program broadcasted. Realizing the importance of the investigation, the broadcasting station scrupulously maintains a constant energy output in its antenna current, and each night before the observers begin work the receiving set is carefully calibrated by means of a small local oscillator in the laboratory placed in close proximity to the receiving set. The output of the local oscillator necessary to maintain full deflection of the recorder in the receiving circuit is then read from the micro-ammeter in the circuit. The constant of the apparatus for the evening is thus determined. In this way local sources of error, both at the broadcasting and receiving ends, are eliminated and the resulting measure of the variable reception from night to night may be attributed to the changing electrical conditions of the atmosphere through which the broadcasted wave travels enroute from Chicago to the receiving station.

Opinions differ as to just what happens when a broadcasted wave travels over the earth.. Some believe that an ether wave is propagated which is reflected back to earth from an ionized layer of the earth's atmosphere, known as the Kennelly-Heaviside layer, which lies some seventy kilometers above the earth's surface. Others maintain that the electric wave is refracted rather than reflected from such a layer. Whatever the mechanism, the wave appears to be turned back by this ionized layer of the earth's atmosphere. Any change in the intensity or degree of this ionization or electrification of the earth's upper atmosphere would have the effect of bending the ray more abruptly or less abruptly towards the earth and thereupon at once be noticed in the intensity of radio reception. The more rapid changes of this sort are doubtless responsible for the phenomena of fading, with which every radio fan is thoroughly familiar. According to our theory the sun constantly bombards the earth's atmosphere with electrons or bundles of energy of high frequency which in turn tear apart the positive and negative charges of the atmospheric molecules, in other words, ionize it to a very considerable extent, thus producing the Kennelly-Heaviside layer.

If the sun is more active on occasion, as when large spots appear on its surface, the degree of ionization increases, producing substantially the effect of lowering the Kennelly-Heaviside layer and upsetting the radio reception. When the sun is again less active, the atmosphere tends to return to its normal state of ionization and the radio broadcasting reception tends to improve as the ionized layer lifts.

For certain wave-lengths it is possible that the effect of a rising and falling ionized layer may actually be the reverse of that noted in the broadcasting zone, giving improved reception during less solar activity. Curiously enough this is just what has been observed by Dr. Pickard at the Newton Centre laboratory when working on long waves of eighteen kilocycle frequency.

FIG. 14.



Graph of Wolfer Sunspot Numbers (3 month running means) showing 15 month fluctuation in rising solar activity since last minimum in 1923.

Further study of the data shows a definite fourteen or fifteen month period in solar activity to be exhibited both in the matter of sunspots and in radio reception.

Another important result of the study of the reception curve is to show how completely unfounded is the popular impression that radio reception is universally poor in summer and good in winter. Generally speaking, reception should be better in the winter months on account of the shortened days and decreased daylight. On the other hand, the sunspots and radio curves of 1926-1928 show that the increased solar activity gave much poorer reception in the winter months of both 1926 and 1927 than during the summer months of the same years. Conditions again improved in 1928, but reception again became poor in the fall and winter of 1929. It may be mentioned that the high degree of static due to thunderstorms in the summer months results in the fact that the average radio listener will decrease the sensitivity of his set in summer to lessen these disturbances with the necessary accompaniment of low audible intensity of distant stations. Hence the general impression of a low intensity accompanying warm weather temperature.

The radio reception registered in 1929 has tended to follow the same fifteen-month cycle in the sunspot numbers with a marked depreciation during the recent fall maximum, when, under normal conditions, radio reception should have been improving with the decreasing hours of sunshine.

Some progress has been made by Dr. Pickard and others in the correlation of the temperature changes with radio reception and while concomitant variation markedly exists it is doubtful if the relation is one of cause and effect. It seems far more plausible that changes in the solar activity are more directly responsible for variations in the signal strengths received than that such should be dependent upon any absolute values of atmospheric temperature.

In view of the fifteen-month period of secondary maxima and minima which appears to be definitely located both in the sunspot and radio curve, we looked for another secondary maximum of solar activity in September or October, 1929.

The rise in sunspot numbers last fall corroborated to a remarkable degree the evidence I ventured to present at the New York meeting of the American Association for the Advancement of Science in 1928, that the period of maximum for the present eleven-year cycle had not been passed. Forecasting on the basis of the fifteen-month cycle, which has worked so effectively during the last few years, the year 1930 should show a general decrease in sunspot numbers as the year waxes, with a corresponding increase in radio signal strength in the broadcast zone. By the very end of 1930 and the beginning of 1931, the general rise of a secondary sunspot maximum may be evident. By 1931, however, it is believed we shall be so far from the maximum of the eleven-year period that the secondary maximum period will have no such marked effect upon radio reception and allied electro-magnetic phenomena as have the sunspot maxima of 1928-1929. The general lifting of the ionization level in the earth's atmosphere may be expected to continue with fluctuations through the next six years, but in 1934 solar activity should be as quiescent as at the last minimum in 1923.

Perhaps the most remarkable results of our correlation study has been the discovery that radio apparatus has become an effective tool in the study of solar radiation. Furthermore, since meteorological changes are correlatable with changes in radio reception, it is but fair to specify that a new method has been evolved which may ultimately lead to important correlation between sunspots and the weather. To this end researches will be continued in these closely related lines at the Perkins Observatory. Grateful acknowledgement is due the American Academy of Arts and Sciences for grants from the Rumford fund to aid in the purchase of apparatus for this new field of research in radiation.

In conclusion, it may be said that investigations in radio transmission, together with researches in the change in the earth's magnetism and electricity and the ultra-violet radiation of the sun, may yet prove to furnish the most definite data as to changes in the sun itself.

Some Recent Advances in Toxicology. MAX TRUMPER (*International Clinics*, 1930, [40], 1, 233–246) stresses the relationship of toxicology to industrial hazards and to preventive medicine. The following selections from his comprehensive review of recent progress in this field are of general interest.

Conclusive proof of intoxication by alcohol is not given by the power of the expired air to reduce a dilute solution of potassium dichromate. The alcohol content of the urine, blood, and spinal fluid should be determined. Unbalance is produced by the presence of 0.25 per cent. alcohol in the brain. Other intoxications, e.g., that produced by carbon monoxide, may be mistaken for alcoholic intoxication.

Leakage of certain refrigerants (ammonia, sulphur dioxide, methyl chloride) used in mechanical refrigerators in homes and apartment houses may be a source of danger. Ammonia and sulphur dioxide betray their presence by their irritative action on the respiratory passages.

Lead has a metabolism similar to that of calcium. The modern treatment of lead poisoning first uses a diet and medication favoring deposition of calcium in the bones; this also favors withdrawal of the lead from the circulation and its deposition in the bones. Then the diet and medication are changed, in order to produce a gradual withdrawal of the lead from the bones and its elimination.

Carbon tetrachloride is used extensively in the industries. It injures the liver. A diet high in carbohydrates and administration of calcium salts are of value in cases of poisoning by this compound.

Among the hazards to which firemen are subjected are the common products of combustion, irritating smoke from fuel oil, gas from leaks in pipes, refrigerants, and carbon tetrachloride vapors from fire extinguishers. A universal gas mask is the best protection from the action of these poisons.

Benzene and certain coal-oil products are used extensively as motor fuel, solvents, etc., and may act as poisons, e.g. by inhalation of their vapors.

J. S. H.

THE DIELECTRIC CONSTANT OF BROMINE VAPOR.

BY

ARTHUR BRAMLEY, Ph.D.,
Assistant to Director, Bartol Research Foundation of The Franklin Institute.

BARTOL RESEARCH FOUNDATION
Communication No. 49.

The dielectric constant of bromine (JOUR. FRANK. INST., July 1926; Feb. 1927) was measured by two different methods, (i) a high frequency bridge and (ii) a direct current bridge method with the aid of an electrometer. The first method gave values for the dielectric constant which varied linearly with the pressure at low pressures but increased abnormally at higher pressures. These results, which are similar to those found by C. T. Zahn for water vapor, were subsequently shown by Wolf (*Phys. Zeit.*, **27**, 588) to arise from the conduction of the liquid condensed on the condenser. The second method gave results at a temperature of 28.5° C. which was independent of the applied electric field and varied linearly with the pressure. However, at higher temperatures and low pressures, the dielectric constant varied with the field strength and was considerably larger than its values at the lower temperature of 28.5° C.

The data for the dielectric constant D measured at 28.5° C. by the electrostatic method give the value

$$D = 1.0047 \pm .0003$$

reduced to standard conditions of 760 mm. pressure and 0° C.

The values of the dielectric constant at 23.9° C. as measured by the high frequency method for low pressures, i.e., pressures less than those for which the conductivity of the cell caused a fictitious increase in the measurements of the dielectric constant, was

$$D = 1.0046 \pm .0003$$

reduced to standard conditions.

The values of the dielectric constant at the higher temperatures are considerably higher than those given above, and also depend on the voltage applied between the condenser plates. Although the voltage difference between the condenser plates was only of the order of several hundred volts, the electric gradient was of the order of several thousand volts per cm. The theory, outlined in the paper referred to above, that at the higher temperatures the molecules were raised to a higher energy level from the normal state by heat radiation gave a satisfactory explanation of the results.

This assumption leads to an expression for the dielectric constant D at a temperature T and a pressure P of the form

$$\frac{D-1}{D+2} = P \sum_{s=1}^{\infty} \sum_{p=1}^{\infty} B_s^p \, e^{\frac{h\nu(sp)}{kT}} \lambda^2 sp,$$

where B_s^p is proportional to the transition probability of a transition from the state s to some other state p and $\nu(sp)$ is the absorption frequency associated with this transition, a negative number in the case of absorption of energy.

The dielectric constant can vary with the electric field according to this theory either from a change in the value of the transition probability or from a pressure change. The experimental results show that in the higher energy levels the electric moment is much larger than in the normal state and hence we should expect a relatively larger change in the dielectric constant with field strength at the higher temperature. Thus the values of the dielectric constant for the lowest potential difference will give the most accurate estimate of the dielectric constant without correcting the pressure for the electrostriction effect.

The data give for the dielectric constant at standard conditions the values in the following table:

Pressure.	Temperature.	$(D-1) \times 10^{-3}$.
126	77	8.5
295	82	6.5
144	120	11.1

From this table we see that the dielectric constant increases with the temperature but decreases with pressure and that the values are approximately double those at the lower temperature of 28.5° C. This indicates that the electric moment of the molecules is greater at the higher temperatures which correspond to higher energy states. Since the dielectric constant decreases with increasing pressure the molecules which are raised to the higher energy levels by the absorption of radiation must lose this energy on collision with other molecules and revert to the normal state. The radiation which the molecules absorb may be either the black body radiation corresponding to the temperature of the gas or it may be higher frequency radiation emitted by the glowing heating coils in the furnace inclosing the condenser.

Measurements of Miss Annie Z. Anderson (*Phy. Soc. of London*, **40**, 62) give a value for the dielectric constant of liquid bromine of 3.11 at 15° C. If we use the relationship that $D - 1/D + 2$ is proportional to the density at a given temperature, then we find for the dielectric constant of gaseous bromine at 15° C. under standard conditions

$$D - 1 = 2.9 \times 10^{-3}$$

a value which agrees fairly closely with ours of

$$D - 1 = 4.7 \times 10^{-3} \text{ at } 28.5^\circ \text{ C.}$$

C. T. Zahn has shown that acetic acid vapor shows an anomolous behavior at higher temperatures and attributes this to the change in the molecules from the normal state to a higher energy state with increasing temperature (*Phys. Rev.*, **35**, 1047).

These experiments on acetic acid vapor lead to the same conclusion which we found in the case of bromine vapor that the dielectric constant of gaseous molecules can be changed through the absorption of infra red radiations raising the molecule from the normal state to some excited state.

Experiments on Modifying Radioactive Disintegration. F. BEHOUNEK. (*Phys. Zeit.*, March 1, 1930.) Miss Maracineanu two or three years ago in Paris believed that she had demonstrated a marked effect of sunshine on the radioactivity of uranium, lead, zinc and copper. The metals she examined formed a part of the roof of the Paris Observatory and the sides on which sunlight fell were, according to her results, more radioactive than the sides permanently in shadow. The author repeated the experiments in Prague and found no effect at all. He further tested the results of experiments by Bogojavlensky who kept for a period of 5 months a polonium preparation at each of 18 different places in U. S. S. R. He measured for each the time required for half of the radioactive material to disintegrate and found it to vary according to locality from 125 to 181 days. When all the specimens were assembled at Leningrad the times ranged only from 137.2 to 139.5 days. He inferred the presence to a varying extent at different places of an exciting radiation proceeding from the interior of the earth. Behounek put polonium preparations in a shaft 813 meters below sea level, and at elevations of 200 and 1332 meters above the sea. No change in the rate of disintegration could be detected.

A uranium preparation emitting beta rays was kept in a mine but no change in its action was found. All the results go to show that it is difficult to modify at will the process of disintegration.

G. F. S.

SCHRÖDINGER DYNAMICS.

BY

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BARTOL RESEARCH FOUNDATION
Communication No. 50.

Due to an arithmetical error in the paper on "Schrödinger Dynamics" (JOUR. FRANK. INST., Nov. 1928), a factor 2 was omitted from the solution of the equation of motion of a free particle. Since this omission might cause some difficulty in deriving the results stated in that paper, we shall briefly sketch the correct derivation.

The equation of motion of a free electron can be written in the form:

$$\frac{\pm h}{2\pi i}\frac{\partial p}{\partial x} + p^2 - 2mE = 0,$$

whose general integral is

$$\log \frac{p - \sqrt{2mE}}{p + \sqrt{2mE}} = \sqrt{2mE}\left(\frac{4\pi i}{h}x + c\right),$$

where c is a constant of integration, or

$$\frac{p - \sqrt{2mE}}{p + \sqrt{2mE}} = e^{\frac{4\pi i}{h}\sqrt{2mE}\,(x+x_0)},$$

which gives for p the value

$$p = \sqrt{2mE}\left\{\frac{1 + e^{\frac{4\pi i}{h}\sqrt{2mE}\,(x+x_0)}}{1 - e^{\frac{4\pi i}{h}\sqrt{2mE}\,(x+x_0)}}\right\},$$

where the sign of the $\sqrt{2mE}$ must be chosen such that p is a continuous function of x.

In case x_0 is real we can write

$$p = -i\sqrt{2mE}\cot\frac{2\pi}{h}\sqrt{2mE}\,(x + x_0).$$

In the region from

$$\frac{2\pi}{h}\sqrt{2mE}\,(x + x_0) = 0$$

to

$$\frac{2\pi}{h}\sqrt{2mE}\,(x + x_0) = \pi$$

the square root signs can be taken both positive for which choice p decreases continuously from minus infinity to plus infinity. In order to make the momentum p vary continuously in the region from

$$\frac{2\pi}{h}\sqrt{2mE}\,(x + x_0) = \pi$$

to

$$\frac{2\pi}{h}\sqrt{2mE}\,(x + x_0) = 2\pi,$$

the sign of the first square root of $2mE$ must be taken as negative while the sign of the second square root of $2mE$ remains unchanged or, expressed analytically, p must be written in this interval in the form

$$p = i\sqrt{2mE}\cot\frac{2\pi}{h}\sqrt{2mE}\,(x + x_0).$$

This arbitrariness in the signs of the $\sqrt{2mE}$ is legitimate since they arise in different ways in the solution of the differential equation.

Thus the momentum p is a periodic and continuous function of x with period

$$\lambda = \frac{h}{\sqrt{2mE}}$$

the same result as stated in the original paper. The case where the constant "c" is real or complex can be handled in a similar manner and leads to the same conclusions regarding the periodicity of the momentum "p" as a function of the spatial coördinate x.

THE COMET TAIL BANDS OF CARBON MONOXIDE.

BY

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Bartol Research Fellow.

BARTOL RESEARCH FOUNDATION
Communication No. 51

The comet tail bands found in the spectrum of Comet Morehouse were first obtained in the laboratory by Fowler[1] who produced them by cathode ray bombardment of carbon monoxide at low pressures of .01 to .005 mm. The spectrum appeared as broad double lines with slight shading toward the red. Since then, these bands have been studied by numerous investigators[2] and it has been established that the comet tail bands arise from positively charged carbon monoxide ions. This conclusion, although quite convincing, nevertheless involved an indirect method of analysis which consisted of identification of these bands as due to CO^+ from relationships calculated by Birge and Sponer between the molecular constants of CO and CO^+, and also from the fact that they belong to the negative groups of CO.[3]

The present paper deals with a direct method for determining whether the emitters of the comet tail bands are positive or negative ions or neutral molecules. This method can be applied in general to any band system, the only restriction being that the average life of the level concerned be of a sufficiently long duration.

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[1] *Mon. Not. Royal Astron. Soc.*, **70**, p. 176, 1909; **70**, p. 484, 1910.

[2] Merton & Barrett, *Phil. Trans. Roy. Soc.*, 222A, p. 386, 1922. Merton and Johnson, *Proc. Roy. Soc.*, 103A, p. 383, 1923. Hulthén, *Ann. d. Physik*, **71**, p. 41, 1923. Johnson, *Proc. Roy. Soc.*, 108A, p. 343, 1925. Lemon, *Proc. Nat. Acad.*, **11**, p. 41, 1925. Baldet, *Compt. Rend.*, **180**, p. 820, p. 1201, 1925. Birge, *Nature*, **116**, p. 171, p. 207, 1925; **117**, p. 229, 1926; *Phys. Rev.*, **28**, p. 1157, 1926. Birge and Sponer, *Phys. Rev.*, **27**, p. 641, 1926. Duffendack and Fox, *Astrophys. J.*, **65**, p. 214, 1927. Hepburn, *Phys. Rev.*, **29**, p. 212, 1927.

[3] Bands arising from positive ions will appear most intense near the negative electrode in a discharge tube, therefore they are called "negative groups."

EXPERIMENTAL ARRANGEMENT AND PROCEDURE.

The experimental arrangement used was quite similar to that described by the writer[4] in the work on the measurement of the average life in the excited state for some of the mercury spark lines. The essential parts of the apparatus are shown diagramatically in Fig. 1. The filament *F* served as a source of electrons. It consisted of a thin platinum strip 0.001 inch thick and three millimeters wide, with a barium and strontium oxide coating. The electrons were accelerated through the

FIG. 1.

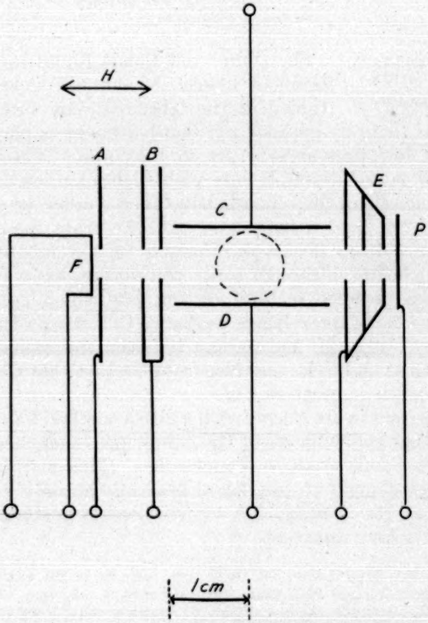

Diagram of electrodes and filament which were mounted in the experimental tube.

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[4] *Phys. Rev.*, 32, p. 715, p. 721, 1928.

slit system by the application of a difference of potential between the plate *A* and the pair of plates *B*. The plate *A* screened off part of the surface of the filament so that only those electrons emitted by the central portion of the filament were sent through the openings in *B*. The filament was usually kept at the same potential as the plate *A*. The opening in *A* was slightly smaller than the openings in *B*. An external magnetic field in the direction indicated at *H* (of the order of 1000 gauss) served to confine the electrons into a beam, so that they never actually touched the plates *B* in moving along their paths. The size of the beam was determined by the size of the opening in *A*. The electrons travelled between the parallel plates *C* and *D* through the slit system *E* and finally were collected by the plate *P*. All of these electrodes were made of molybdenum because of its non-magnetic property and because it gave off very little occluded gas. The parts illustrated in the figure were mounted in a bulb which contained the gas to be investigated, at a pressure between 1×10^{-2} mm. to less than 1×10^{-4} mm. The electron current to *P* was usually kept at approximately 1/10 of a milliampere. There was applied an additional field between the parts *E* and *P* to prevent the return of secondary electrons from the plate *P*. A cross-electric field was applied between the plates *C* and *D* for the purpose of drawing out the ions formed in the beam at right angles to the direction of the electron beam. The ions because of their large mass were not appreciably affected by the magnetic field. The actual path of the ion was a cycloid of a large diameter so that the ions moved in practically straight lines while being accelerated to the side plate. The electron beam was not appreciably altered by the application of the cross-electric field.

The light coming from the region between the parallel plates indicated by the dotted circle was photographed by a quartz spectrograph. The light excited by the electron beam was focused by means of a quartz lens on to the slit of the spectrograph so that the direction of motion of the ions was parallel to the length of the slit. Thus we have a situation in which the emitter of a band will be moving in a particular direction if it is a charged molecule, hence producing a variation of the intensity, or displacement, of the band along its length as found on the spectrogram. In the case where the emitter is a neutral molecule the action of the electric field will have no effect on the intensity distribution of the band system. Therefore it is possible by this means to differentiate between bands produced by ions from those produced by neutral molecules. However, a case might arise in which the bands radiated by a charged ion would exhibit no displacement and hence be erroneously classified as due to a neutral molecule. This would occur if the time that the ion remained excited was so short that it would become de-excited before having an opportunity to move appreciably under the influence of the electric field. In this case the intensity distribution of the band along its length would appear practically the same in the presence of the electric field as in its absence. This fact would lead one to believe that the band was emitted by neutral molecules.

In an arrangement of this kind there are several actions which might occur. (1) A normal unexcited molecule could be simply ionized without being excited. Immediately the ion would move under the influence of the electric field, but before being swept out completely from the region of the beam it could suffer a second collision with an electron of the beam causing it to be put in the excited state. Then later, when it had moved over to a new position it would radiate its characteristic light. A phenomenon of this kind would appear on the photographic plate as a displaced band partially extending into the region outside of the electron beam, and indicating correctly that the band is emitted by an ion. The amount that the band would be displaced would depend on the probability of excitation of the ions by the electrons of the beam, and also on the average time that the ions remained excited. (2) The second possibility also involves two separate events. An unexcited ion is formed by a collision with one of the electrons of the beam, and then the ion moves under the influence of the field but before leaving the region of the beam it recombines with a free electron forming an excited neutral molecule. This molecule in its displaced position then radiates light which is characteristic of the un-ionized, rather than of the ionized, molecule. The bands would appear displaced and hence appear erroneously as being emitted by charged molecules. This case although possible, is ruled out because it is quite improbable that ions would recombine in sufficient numbers at least to give appreciable light intensity. It is found from the work of Mohler [5] that recombination spectra are difficult to obtain and only do occur in regions where there exists a high concentration of ions and slow moving electrons, a situation which does not exist in the present arrangement. (3) An excited ion is formed at the first collision and then the ion moves an appreciable distance before radiating, giving the displaced band in the spectrum. This is the type of action found to occur by the writer [6] in the case of spark lines and is assumed to be the most probable process to take place in the present arrangement.

The order of magnitude of the ion current to side plate *C* was generally about one tenth of the primary electron current in the beam. That is, of the order of 1×10^{-5} amperes at 1×10^{-3} mm. pressure. The amount of this current, of course, increases proportionally to the pressure in the tube. The potential difference between the electrodes *A* and *B* was kept at approximately 150 volts. This was sufficient to give the electrons the necessary velocity to efficiently ionize and excite the carbon monoxide molecule. The difference of potential between electrodes *E* and *P* gave the electrons an additional velocity of 150 volts. That is, the electrons would be first accelerated to 150 volts velocity, then receive an additional velocity of the same amount just before striking the plate *P*, making their final velocity on reaching the plate equal to that corresponding to 300 volts. It was found in general that the velocity of the secondary electrons produced by the 300 volt electrons was less than 150 volts so that the secondary electrons were unable to return to the region between the parallel plates *C* and *D*. It was found necessary to eliminate the secondary electrons from this region because their presence caused the boundary of the beam to become destroyed.

Since other investigators [7] have found the comet tail bands to be excited in mixtures of CO and helium, provision was made for the circulation of helium through the apparatus. This was accomplished by connecting in series with the tube a mercury diffusion pump with liquid air traps and a reservoir containing helium together with suitable leaks so that the helium could be used over again after being purified in the reservoir. A glow discharge between an iron and a misch metal electrode at about 5 mm. pressure served to purify the helium. This method of handling the gas has been described by Compton and Boyce.[8] However in the experimental tube there remained sufficient traces of carbon monoxide as an impurity to cause the comet tail bands to appear sufficiently intense for photographing without having to introduce carbon monoxide directly into the system.

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[5] *Phys. Rev.*, **31**, p. 187, 1928.

[6] *Phys. Rev.*, **32**, p. 721, 1928; **34**, p. 199, 1929.

[7] Merton and Johnson, ref. cit.; Merton and Barrett, ref. cit. Hulthén, ref. cit. Baldet, *Compt. Rend.*, **180**, p. 820, 1925. Johnson, ref. cit. Lemon, ref. cit.

RESULTS AND DISCUSSION.

In Fig. 2 there are reproduced prints from the actual negatives obtained with several hours exposure, which illustrates the appearance of the comet tail bands. The bands appear in pairs with a degrading of intensity toward the red which is in agreement with their description as given by Fowler. In these cases the helium was not admitted to the tube and the pressure was as low as could be obtained without baking out the glass walls or removing the vapors arising from stop cock grease and from traces of paraffine on the filament leads. The pressure in the tube as indicated by a McLeod gauge was less than 1×10^{-4} mm. of Hg. The spectrogram No. 1 has superimposed on it a copper arc spectrum used for the purpose of measuring the wave-lengths of the bands. It also has placed directly above it a wave-length scale which locates approximately the wave-lengths of the bands. The bands may be seen more easily in photograph No. 2 of Fig. 2, in which they are reproduced without the comparison spectrum and wave-length scale. In these cases the cross-electric field was not applied so that the bands are not displaced and their length simply shows the width of the beam.

For the purpose of identifying the bands their wave-lengths were determined and the results are shown in Fig. 3. The measured wave-lengths of the bands are given in the first column. The resolving power of the spectrograph was not great

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[8] JOUR. FRANK. INST., **205**, p. 497, 1928.

FIG. 3.

Observed Wave Length of Band.	Wave Number of Head.	ν (Origin).	n'.	n''.	$\Delta\nu$.	$\Delta\lambda$.
4712.4 to ——	21214	21207	2	1	7	− 1.6
4690.3 to ——	21314	21333	2	1	19	+ 4.3
4565.9 to 4579.9	21895	21896	1	0	1	+ .2
4543.8 to 4558.1	22001	22021	1	0	20	+ 4.0
4271.8 to 4288.2	23402	23390	2	0	12	− 3.2
4250.5 to 4262.4	23520	23515	2	0	5	− .9
4017.8 to 4028.2	24882	24870	3	0	12	− 2.0
3999.2 to 4010.8	24997	24995	3	0	2	− .3
3796.3 to 3806.7	26333	26322	4	0	11	− 1.7
3779.6 to 3795.7	26450	26447	4	0	3	− .4
3602.0 to 3608	27754	27746	5	0	8	− 1.0
3586.6 to 3594.0	27873	27871	5	0	2	− .3

Table of measured and calculated wave-lengths of the comet tail bands.

enough to separate the components of the bands so that the figures in the table give the region of wave-lengths over which the band occurs. The wave-length of the head of each band is given by the wave-length at which the band begins, i.e., given by the first column of figures in the observed wave-length column. The next column contains the corresponding wave numbers for these band heads. The third column gives the wave number of the origin for the comet tail bands as computed by the usual band spectrum formula:[9]

$$\nu_0 = \nu_e + n'(\omega_0' - \omega_0'x'n' + \cdots) \\ + n''(\omega_0'' - \omega_0''x''n'' + \cdots)$$

In which ν_0 is the wave number of the origin of the band, ν_e the spacing of the two electronic levels involved, while the expression $n(\omega_0 - \omega_0 xn + \cdots)$ represents the vibrational energy. The values of the constants, ν_e, ω_0', $\omega_0'x'$, w_0'' and $\omega_0''x''$ were obtained from Birges tables in the International Critical Tables.[10] The values of the quantum numbers n' and n'' are given in the table. The last two columns contain the difference between the experimentally determined values for the heads and the computed values for the origins. This difference is expressed both in wave numbers and in wave lengths (Ångstroms). The agreement is quite satisfactory, in fact good enough to establish the identification of these bands as the comet tail bands.

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[9] Report of the National Research Council on "Molecular Spectra in Gases," p. 122.

[10] International Critical Tables 5, p. 409.

In the photographs represented in Fig. 2, there is seen a pair of faint wide lines indicated by the two black dots. Their wave-lengths as measured from the center of each of the lines were found to be 2883Å and 2896Å. The accuracy of measurement was to within about one Ångstrom. These lines are identified as bands belonging to the first negative group of Deslandres, because their wave-length is practically the same as that of two very prominent bands of this group, which are located at 2883.8Å and 2897.1Å.[11]

By circulating the helium through the tube at a pressure of 1×10^{-3} mm. both the helium spectrum and the comet tail bands are obtained as is shown in Fig. 4. Both spectra appear with about equal intensity. In the upper spectrogram the cross-electric field was applied so that the positive ions were moving in the direction indicated by the arrow. The two comet tail bands which occur in the neighborhood of 4300Å indicated by the dot are actually displaced in the direction of motion of the positive ions. This can be detected most easily by noting the position of the bands relative to the line of the wave-length scale which represents 4300 in the upper photograph, as compared with the lower which was taken in the absence of the electric field. To illustrate this "displacement effect" more clearly we have shown in Fig. 5 a reproduction of a densitometer record taken for one of these bands. The record shows the variation of intensity of the band along its length and also its position relative to the electron beam. The positive ions were moving in the direction indicated by the arrow. Curve 1 is a record for the comet tail band, while curve 2 is a record for an undisplaced helium arc line which gives the position of the electron beam. It is easily seen that the band has been displaced in the direction in which the positive ions were moving. Without the presence of the electric

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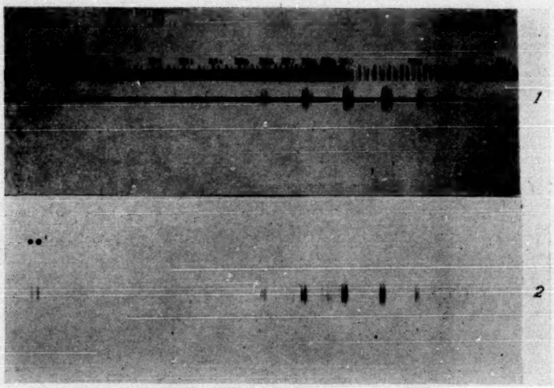
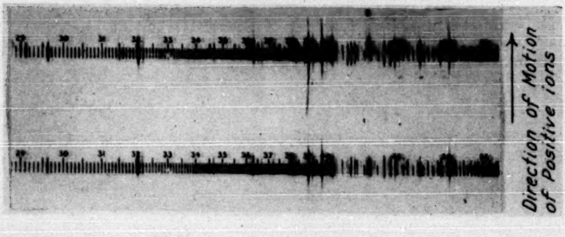
[11] These are the wave-lengths given in Kayser's "Handbuch der Spectroscopie," vol. 5, p. 233. Also measured later by Johnson, ref. cit.

FIG. 2.



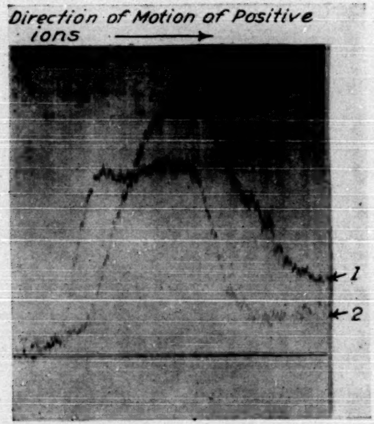
Reproduction of prints from the negatives obtained illustrating the appearance of the comet tail bands and two bands of the first negative group of Deslandres.

FIG. 4.



Reproduction of a print from the negative obtained illustrating the displacement of the comet tail bands (indicated by the dot) in the upper spectrogram. In the lower photograph with the cross field absent, the bands are undisplaced.

Fig. 5.



Densitometer record showing the displacement of the comet tail band in record No. 1, while No. 2 is a similar record for one of the helium arc lines which gives the location of the electron beam.

field the band would have the same position and shape as the helium arc line, that is the same as curve 2. The other prominent comet tail bands showed a displacement similar to the above. This proves that the comet tail bands are due to positively charged ions, which is in agreement with the conclusions made by Birge and others as regards their origin.

The two bands at 2883Å and 2896Å of the first negative group did not exhibit the displacement effect when the cross field was applied. This is interpreted as showing that the average life time of the excited state of the CO^+ molecule which produces these bands is so short that the ion does not move appreciably under the influence of the electric field before it becomes de-excited. Consequently the mean life of the CO^+ ion for the emission of these bands is considerably shorter than for the emission of the comet tail bands.

When the pressure of helium in the tube was increased to 2.8×10^{-3} mm. the comet tail bands disappeared leaving the helium spectrum. The two first negative group bands became weakened but were still visible on the plates. Other investigators[12] have found, on the contrary, that helium at a pressure between 1.5 to 6 cm. containing slight traces of impurities of carbon monoxide, is capable of brilliantly emitting the comet tail bands. When the helium pressure is high there is a large concentration of helium atoms in the metastable state of energy above 19 volts. Since the energy necessary to excite the comet tail spectrum is about 17 volts[13] a large number of collisions of the second kind with the carbon monoxide molecules could increase the number of excited CO molecules and consequently produce an increase in the intensity of the comet tail bands. However in the present work the concentration of excited helium atoms was relatively small so that there would be a small amount of transfer of energy by collisions of the second kind. To explain this diminution of intensity of the bands one resorts to the following considerations; when the pressure of helium is increased to 2.8×10^{-3} mm. the flow of gas through the tube becomes greater and the impurities are carried out more rapidly than when the pressure and rate of flow is less. At this higher pressure therefore, a lower concentration of CO molecules would exist and consequently the intensity of the bands would be diminished.

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[12] See references given in footnote 7.

[13] Birge, *Phys. Rev.*, **28**, p. 1157, 1926. Duffendack and Fox, ref. cit.

Since it has been previously found that the first negative group and the comet tail bands are very sensitive to slight changes in pressure and to the presence of foreign gases, it seems probable that the cause of this diminishing of intensity of the bands may be related to these previously obtained results,[14] and that the above explanation may be insufficient.

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[14] Baldet, *Compt. Rend.*, 180, p. 1201, 1925, and references in footnote 7.

SLOT LEAKAGE IN A.C. GENERATORS.

BY

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When there is no current in the conductors of a slotted armature, the magnetic flux in the air gap passes through the teeth into the armature core and is "cut" by that portion of the armature winding which is buried in the slots. Even if the flux density in the teeth is very high, causing an appreciable part of the total flux to pass into the core by way of the slots, this does not alter the fact that, under open-circuit conditions, all the flux in the air gap under the poles is cut by the conductors and thus causes an e.m.f., proportional to the air-gap flux, to be developed in the armature windings. But when current is drawn from the armature, a magnetomotive force proportional to the ampere-conductors in a slot, will cause some flux to be diverted from tooth to tooth across the slot, and this flux will not be cut by all the armature conductors.

The effect of the slot leakage flux is to cause a change in the number of flux lines "cut" by the active conductors, and the e.m.f. developed in the armature winding is not the same as it would have been if all the flux lines passing from pole face to armature had continued down into the core through the roots of the teeth or by a parallel path through the slots.

With few exceptions,[1] textbooks treat this problem in electrical machine design as if there were an actual interlinkage of slot leakage flux and armature conductor, which could be considered as producing an e.m.f. of self induction in all respects similar to other self-induced e.m.fs., such as the IX

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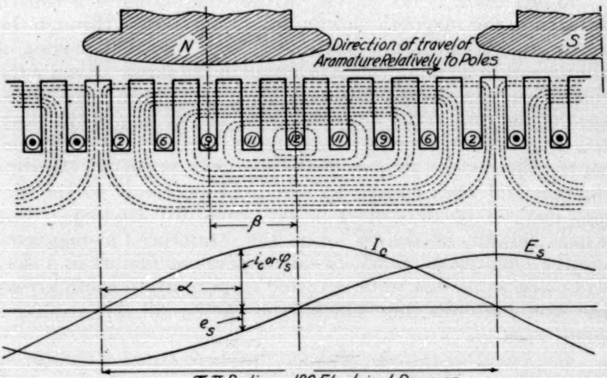
[1] C. C. Hawkins, *The Dynamo*, Vol. III, sixth edition, 1925. Isaac Pitman & Sons, Ltd., p. 123 *et seqq.* S. P. Smith, *Papers on the Design of Alternating Current Machinery*, 1919, Sir Isaac Pitman & Sons, Ltd., p. 9. Alfred Still, *Elements of Electrical Design*, McGraw-Hill Book Co., Inc., p. 370.

drop due to the self inductance of the overhanging portion of the armature windings which form the end connections between the "active" conductors in the slots.

In this article, the writer will attempt to give a physical interpretation of what occurs in slotted armatures having appreciable slot leakage, from which it will be seen that what is commonly referred to as an *IX* drop due to slot leakage is not a true e.m.f. of self inductance and should not be so considered.

FIG. 1.


N
S
Direction of travel of Aramature Relatively to Poles
2
6
9
11
12
11
9
6
2
β
I_c
E_s
i_c or φ_s
α
e_s
τ=π Radians=180 Electrical Degrees

The dotted lines in Fig. 1 form a diagrammatic representation of the slot flux in the armature of a synchronous a.c. generator, assuming that this flux could be considered independently of the main flux entering the armature from the pole faces. Such a condition does not exist in an actual machine operating under load, and it could only be realized by passing current through the armature windings from an independent source while the machine is driven at normal speed with the field circuit disconnected. It is assumed that all the slot flux set up by the conductors in any one slot entirely surrounds these conductors, that is to say, all flux lines cross the slot in the space above the conductors, and the effect of leakage flux passing through the material of the conductors is, for the present, neglected.

Another assumption which will be made, not only in connection with Fig. 1, but throughout this article, is that there are as many phases as there are slots in the space of one pole pitch. Thus, the generator of Fig. 1 must be thought of as a nine-phase machine. If the armature were that of a three-phase machine, there would be three slots per pole per phase, and the ampere-conductors in each slot of a group of three adjacent slots would have the same value at a given instant of time. The assumption of n phases in a machine with n slots per pole has been made in order to simplify the problem and justify the use of vector diagrams.

The distribution of current in the several conductors will depend upon the position of the slot relatively to the field system. It will be convenient to consider a stationary field system with rotating armature. The current is supposed to have its maximum value when the angular displacement of the slot relatively to the center line of the pole is β. On the assumption that the current variations follow the sine law, the figures on the diagram represent approximate current values for various slot positions in the space of one pole pitch. Since the slot leakage flux is proportional to the ampere-conductors in the slot, a correct diagrammatic representation of this flux is obtained by making the number of lines which cross from tooth to tooth in each slot proportional to the current in the conductor. The reluctance of the iron in the path of the slot leakage flux is considered negligible relatively to that of the air path between adjacent teeth.

If I_c stands for the r.m.s. value of the armature current, its maximum value is $\sqrt{2}I_c$, and the value of the current in a slot displaced by an angle α from the position where $i_c = 0$ is

$$i_c = \sqrt{2}I_c \sin \alpha \tag{1}$$

and since the slot flux is proportional to this current, we may write

$$\varphi_s = KI_c \sin \alpha, \tag{2}$$

where K is a constant depending upon the slot dimensions and the number of conductors in a slot. The sine curve marked I_c in Fig. 1 may therefore stand for either the armature current or the slot flux for any position of the armature slot relatively to the field poles.

The instantaneous value of the e.m.f. developed in a single slot conductor due to the rate of change of the flux linkages is

$$e_s = -\frac{d\varphi_s}{d\alpha} = -KI_c \cos \alpha, \tag{3}$$

from which the curve marked E_s can be plotted. This is obviously a sine wave lagging behind the sine wave of current distribution by a quarter period.

Bearing in mind that the flux represented diagrammatically by the dotted lines in Fig. 1 may be considered approximately stationary in space while the slotted armature, carrying the conductors, is moving from left to right, the e.m.f. of formula (3), represented by the curve E_s, can be calculated by considering the rate at which the flux lines entering the armature core through the roots of the teeth are "cut" by the conductors in the slots. If it is recognized that the total e.m.f. developed in the armature under actual load conditions is due to the cutting of all the flux lines linking with the slot conductors, it is easy to avoid the erroneous conclusions which result from failure to discriminate between slot leakage flux and the self-induction of the overhanging portions of the armature winding.

In the diagram Fig. 2, the same simplifying assumptions have been made as in connection with Fig. 1, namely that there are as many phases as there are slots in one pole pitch, and that all the slot leakage flux passes from tooth to tooth in the space above the conductors. No attempt has been made to show the actual paths of the flux lines, but the figures on the arrows indicating direction of flux in slots and teeth, are proportional to the number of flux lines entering the teeth from the air gap, passing across the slots and entering the armature core through the roots of the teeth.

It is assumed that the combined effect of field and armature ampere turns is to produce a sinusoidal flux distribution over the armature surface [2] as represented by the curve E_d'. The current distribution in the armature conductors is indicated by the curve I_c which represents not only the current, but also the slot m.m.f. and slot leakage flux. Considering a particular tooth *t*, the fact that the slot ampere-conductors are greater in the adjacent slot on the right than in the adjacent slot on the left leads to a tooth m.m.f. proportional to 154 − 100 = 54 which opposes the m.m.f. tending to send flux through the tooth into the armature. The flux entering the armature core through this tooth is therefore not 336 which it would have been if the slot m.m.f. had not caused more flux to leave the tooth on the right hand side than enters it on the left; but 336 + (100 − 154) = 282, as indicated on the arrow through the root of the tooth. If, therefore, we draw the curve marked E_s of which the ordinates are a measure of the difference between the slot fluxes on the two sides of a tooth, and then add the ordinates of this curve to those of the curve E_d', we obtain the curve E_d which represents the flux entering the armature core through the roots of the teeth, and is therefore also a measure of the e.m.f. actually developed in the armature conductors by the cutting of this flux.

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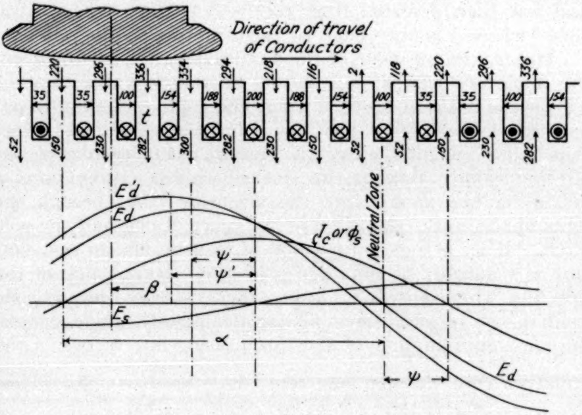
[2] An assumption which may not always be justified in connection with salient pole machines.

FIG. 2.


Direction of travel of Conductors
220
296
336
334
294
218
116
2
118
220
296
336
35
35
100
154
188
200
188
154
100
35
35
100
154
t
52
150
230
282
300
282
230
150
52
52
150
230
282
E'd
Ed
I_c or ϕ_s
Neutral Zone
ψ
ψ'
β
E_s
α
ψ
Ed

It should be noted that the e.m.f. corresponding to the flux curve E_d' has no real existence; it is a fictitious quantity which may be called the *apparent developed voltage*, being the e.m.f. that would have been developed in the armature windings if part of the flux entering the teeth from the air gap had not been diverted from tooth to tooth in the form of slot leakage flux. The difference between the voltages E_d' and E_d is the voltage component that would have been developed if a certain amount of flux—equal to 100 in Fig. 2—had not been diverted from tooth to tooth in the neutral zone[3] where it is not "cut" by the armature conductors.

The maximum value of the slot leakage flux occurs where the armature current reaches its maximum value of $\sqrt{2}I_c$. This is the position in space where the angle α is equal to 90° and the slot leakage flux in Fig. 2 has the value 200. Note that half a pole pitch away, the current has fallen to zero and all the leakage flux in the slot where the current was a maximum has passed into the armature core through the roots of the teeth comprised in the space of one half the pole pitch. If Φ_s = maximum value of leakage flux in one slot, and n = number of slots per pole, the *average* value of the slot flux which enters or leaves the armature through one tooth is $2\Phi_s/n$ and, since we are dealing with sine curves, the maximum amount of slot flux passing into or out of the armature core through one tooth is $\frac{\pi}{2} \times \frac{2\Phi_s}{n} = \frac{\pi}{n}\Phi_s$. This, therefore, is the height of the maximum ordinate of the curve marked E_s in Fig. 2.

The angles ψ and ψ' in Fig. 2 are the angles by which the current lags behind the actual and "apparent" developed voltages E_d and E_d' respectively. It follows that the value of slot leakage flux in the neutral zone, where $\alpha = (180° - \psi)$, is:

$$\text{Slot flux in neutral zone} = \Phi_s \sin \psi \qquad (4)$$

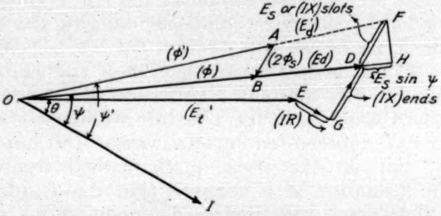
which, for the particular case of Fig. 2, is $200 \times \sin \frac{180 \times 1.5}{9}$ $= 100$ as marked on the arrow across the slot in the neutral zone.

---

[3] The neutral zone is the position in space, or in relation to the field system, where the direction of the flux is parallel to the direction of travel of the armature conductors, i.e., where no lines enter the armature in such a direction as to be "cut" by the conductors.

Vector Diagram Including the Effect of Slot Leakage.—The vector $OE = E_t$ in Fig. 3 represents the voltage at the terminals of an a.c. generator when the armature current, represented by the vector OI, lags behind this voltage by an angle θ.

FIG. 3.



EG, parallel to OI, is the voltage component to compensate for IR drop in the windings.

GD, leading OI by 90°, is the e.m.f. component to compensate for the IX drop due to the inductance of that portion of the armature winding which is outside of the slots and may be considered as beyond the direct influence of the field system.

$OD = E_d$ is the vectorial sum of E_t, IR, and $(IX)_{ends}$. It is the voltage actually developed in the slot conductors, and it leads the current by the angle ψ.

$FD = E_s$ is the e.m.f. component due to the inductance of that portion of the armature winding which is buried in the slots.

$OF = E_d'$ is the *apparent developed voltage* which leads the current by the angle ψ'.

$DH = - E_s \sin \psi$ is the e.m.f. component that would have been developed if slot flux equal in amount to $\Phi_s \sin \psi$ had entered the armature core instead of being diverted from tooth to tooth in the neutral zone.

Considering the several components of the flux, we have;

$OA = \Phi'$ lines entering the tops of the teeth from the field poles in the space of one pole pitch.

$AB = 2\Phi_s$ lines, being slot leakage passing down the teeth into the armature core in the space of one pole pitch. This is the flux component marked E_s in Figs. 1 and 2. Since Φ_s is the maximum leakage flux across one slot, and since we are now considering this flux independently of other flux components, it is necessary, as will be seen in Fig. 1, to take twice this value as the slot flux component entering the armature core in the space of one pole pitch.

$OB = \Phi$ lines, is the vectorial sum of Φ' and $2\Phi_s$ and it is, therefore, the flux actually "cut" by the conductors to develop the voltage E_d in the armature winding.

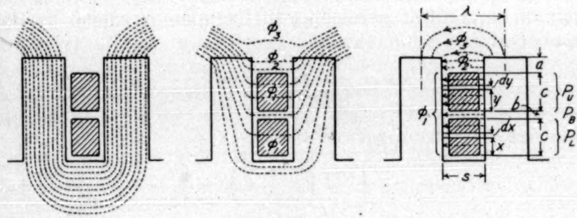
The e.m.f. represented by the vector OF has no real existence, but, in the usual mathematical treatment of alternator regulation, it is assumed that this e.m.f. (E_d') is actually developed and afterwards modified by armature reactance, no distinction being made between the end connection reactance and the slot reactance. No serious error is introduced by assuming the voltage E_d' to be actually developed in the armature provided the slots are not deep and the slot leakage is a small percentage of the total flux per pole. But when the slot leakage is comparatively large, and the flux density in the teeth is high, the errors made in calculating the iron losses and excitation required to overcome tooth reluctance may be appreciable.

Although the diagrams used to illustrate this article show the current lagging behind the terminal voltage, the construction followed in Fig. 3 would be exactly the same for a generator supplying a *leading* current on a condenser load. In this case, the vector OD would be greater than OF, that is to say the m.m.f. which sets up slot leakage flux would cause more flux to enter the core through the roots of the teeth than passes from the air gap into the tops of the teeth, and the *apparent developed e.m.f.* would then be *less* than the e.m.f. actually developed in the armature.

Calculation of Equivalent Slot Flux.—The diagrams Figs. 1 and 2 assume that all the slot flux passes across the slot in the space above the conductors. In practice a large portion of this flux passes through the space occupied by the slot conductors, and, in order to develop a formula for calculating the component FD of Fig. 3 (or determining K in formula (2)), it will be convenient to determine the *equivalent slot flux.* This may be defined as a fictitious flux which, if cut by all the armature conductors, would develop an e.m.f. component equal to the e.m.f. which the actual slot leakage flux would develop if it could be considered independently of other fluxes.

Fig. 4 (*A*) indicates the path of the magnetic lines in the neutral zone when the current in the slots occupying this particular position (relatively to the field system) is zero. Fig. 4 (*B*) is a diagrammatic representation of the flux lines in the neutral zone when the current in the slot conductors has an appreciable value. Note that, of the nine lines entering the tooth top from the air gap, all are "cut" by the slot conductors under the condition (*A*) when there is no leakage flux, but under condition (*B*), the flux lines Φ_2 and Φ_3 are no longer cut by the armature conductors and therefore develop no e.m.f. in the winding. The flux lines Φ_1 which pass through the copper and through the space occupied by insulation between the upper and lower coil-sides of a double-layer winding, are not cut by all the conductors in the slot. Fig. 4 (*C*) illustrates the method of calculating the equivalent slot flux for which the symbol Φ_{es} will be used.

FIG. 4.



(*A*) (*B*) (*C*)

Let C_s = number of conductors per slot, and i_s = current per conductor in amperes; then if M stands for the total slot m.m.f. tending to establish flux leakage across the slot, we have

$$M = 0.4\pi C_s i_s \tag{5}$$

and

$$\Phi_{es} = M \times l_a P_{es} \tag{6}$$

where l_a = gross length of armature core in centimeters, and P_{es} = equivalent permeance per centimeter length of slot. The writer believes that the use of the gross length rather than the net length of the armature core is justified since it includes an allowance for the fringing fluxes at the sides of vent ducts.

The permeance of the flux path of depth dx in Fig. 4 (C) is dx/s, but the amount of the flux in this space is not due to the total m.m.f. M, but to $M(x/2c)$, and this element of flux fails to link with $C_s(x/2c)$ conductors. The *equivalent permeance* of the element considered is therefore $(dx/s)(x/2c)^2$, and the equivalent permeance of the space occupied by the lower coil-side is

$$P_L = \frac{1}{4sc^2}\int_0^c x^2dx = \frac{c}{12s}. \tag{7}$$

Similarly, for the upper coil-side,

$$P_U = \int_0^c \frac{dy}{s}\left(\frac{c+y}{2c}\right)^2 = \frac{7c}{12s}. \tag{8}$$

The equivalent permeance of the space of depth b between the two coil-sides is

$$P_B = \frac{b}{s}\left(\frac{c}{2c}\right)^2 = \frac{b}{4s}. \tag{9}$$

The equivalent permeances of the flux paths Φ_2 and Φ_3 are, of course, the same as the actual permeances of these paths. Using the symbols P_2 and P_3 for these permeances per centimeter of slot length, and summing up the equivalent permeances as expressed by formulas (7), (8) and (9), we obtain for the total equivalent slot permeance

$$l_aP_{es} = l_a\left[\frac{8c+3b}{12s} + P_2 + P_3\right], \tag{10}$$

where l_a is the gross length of the armature core in centimeters.

The equivalent slot flux is therefore

$$\Phi_{es} = 0.4\pi C_s i_s l_a\left[\frac{8c+3b}{12s} + P_2 + P_3\right]. \tag{11}$$

Calculation of Permeances P_2 and P_3.—Assuming parallel sides to the slot in the space of depth a above the coils, we have,

$$P_2 = \frac{a}{s}, \tag{12}$$

which must be modified if the sides of the slot are not parallel at this point. The flux component Φ_3 is supposed to account for the flux lines that cross from tooth to tooth in the air gap outside the slot. Any leakage flux beyond what is included between the center lines of the teeth on each side of the slot should not be treated as slot flux and will therefore be disregarded in these calculations. A convenient formula to use for calculating this portion of the slot leakage permeance in salient pole machines is Finnis' formula which assumes the flux paths to be semi-ellipses with the foci at the edges of the teeth. Putting $r = \lambda/s$, where λ = tooth pitch and s = slot width, as indicated on Fig. 4 (*C*), the tooth-top permeance per centimeter of slot length, for salient pole machines, may be written

$$P_3 = \frac{1}{\pi} \log_3 (r + \sqrt{r^2 - 1}). \tag{13}$$

In machines with cylindrical fields and a uniform air clearance (δ), not only under the pole centers, but in the neutral zone also,

$$P_3 = \frac{\lambda}{\delta}. \tag{14}$$

In order to determine the length of the vector DF in Fig. 3, it is necessary to calculate the e.m.f. component due to the "cutting" of the slot leakage flux by the conductors.

Since $2\Phi_{es}$ is the total amount of the slot flux which, if considered as a separate flux component, enters or leaves the armature core in the space of one pole pitch (see Figs. 1 and 2), the *average* value of the voltage component which would result from the "cutting" of these flux lines is

$$E_{average} = \frac{2\Phi_{es} p N}{10^8 \times 60} (C_s n_s p), \tag{15}$$

where p = number of poles, N = revolutions per minute, C_s = number of conductors per slot, and n_s = number of slots per pole per phase. Or, in terms of frequency, since $Np = 120f$,

$$E_{average} = 4\Phi_{es} f(C_s n_s p) 10^{-8}. \qquad (16)$$

Since we are assuming sine wave shapes, it is necessary to multiply the mean value by $\pi/2\sqrt{2}$ to obtain the r.m.s. value, and if we substitute for i_s in formula (11) its maximum value $\sqrt{2}I_c$, the final expression for the fictitious reactive voltage component per phase due to slot inductance is

$$E_s = I_c \times 0.8\pi^2 f C_s^2 n_s p l_a \times \left[\frac{8c + 3b}{12s} + P_2 + P_3 \right] 10^{-8} \text{ volts.} \qquad (17)$$

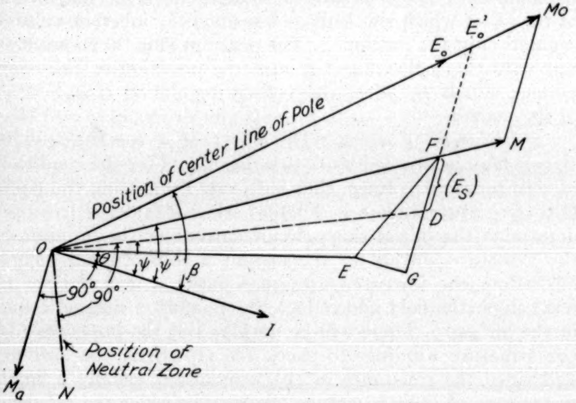
Effect of Slot Leakage Flux on Regulation and Short-Circuit Current.—Since the writer has taken pains to show that the e.m.f. developed in the armature windings, corresponding to a given terminal voltage, is independent of the slot leakage, it may be asked: Why go to the trouble of calculating the "apparent" developed voltage (E_d') when the actual developed voltage (E_d) may be calculated from a knowledge of the resistance of the windings (IR drop) and the inductance of the end connections (IX_{ends}). The answer is that a knowledge of the slot leakage flux is necessary to determine the field excitation required to provide the proper air-gap flux under load conditions, and also to determine the behavior of the machine under short-circuit conditions or sudden fluctuations of load.

The effect of slot leakage flux on voltage regulation is almost identical with that of armature reaction, that is to say, it causes a displacement of the resultant flux relatively to the center line of the field pole, and usually a change in the shape of the e.m.f. wave; but this minor effect is disregarded here since, in order to simplify the treatment, only the fundamental sine waves of the alternating quantities are considered.

The vector diagram Fig. 5 explains how the slot leakage flux is a factor in determining the field excitation required and, incidentally, it will explain how the angle β of Figs. 1

and 2, and the position of the neutral zone with reference to the center of the pole face ($\beta - \psi + 90°$) may be calculated. The assumption of sine wave functions is still made, although, as previously stated, this may not always be justified because of the distortion of the flux distribution curves, especially with salient pole machines.

FIG. 5.



The vectors OI, OE, OD and OF in Fig. 5 have the same meaning as in Fig. 3. The vector OM, in phase with OF (the "apparent" developed voltage E_d') represents the ampere turns required to overcome air-gap and tooth reluctance when the air gap flux is such that it would develop E_d' volts per phase in the armature if it were cut by all the conductors. The vector OM_a, drawn exactly 90° behind OI, represents the maximum value of the armature ampere turns per pole. This must be balanced by the field component MM_0, giving OM_0 as the required field excitation under load. If the load is thrown off, the developed voltage will be OE_0 with the point E_0 on the vector OM_0. The length OE_0 will usually be somewhat less than OE_0', obtained by extending GF until it meets OM_0, because increased saturation of the iron in the magnetic circuit will prevent the developed e.m.f.'s being exactly proportional to the exciting ampere turns.

When the load is thrown off, the maximum value of the voltage wave OE_0 will occur immediately under the center line of the pole face, and β, as obtained from Fig. 5, is therefore the angular displacement from the center line of the pole of the slot conductors carrying the maximum current. The line ON, drawn 90° behind OD, is the position of the neutral zone relatively to the poles. The amount of slot flux passing from tooth to tooth in this zone determines the magnitude of the effect which slot leakage has upon the inherent voltage regulation of the machine. The diagram (Fig. 5) shows how not only the magnitude but also the direction of the component vector E_s affect the voltage regulation $(OE_0 - OE)$ of the machine.

Short-Circuit Current.—The effect of a sustained short circuit may be studied by re-drawing Fig. 5 for the condition of zero terminal voltage, that is to say, by moving the point O to E. The problem is different when it is desired to predetermine the initial short-circuit current or the amount of the armature current at the instant a short circuit occurs, i.e. before the increased armature m.m.f. has had time to react upon the field and reduce the amount of magnetic flux in the air gap. There will be nothing but the impedance of the armature windings to check the growth of the current, and since the resistance of the windings is usually a small percentage of their reactance, we may consider the IR drop to be negligible. It should be observed that the slot reactance which, under normal operating conditions, is correctly treated as a fictitious reactance, now becomes a real reactance similar to that of the end connections.[4] The initial short-circuit current may, therefore, be calculated by assuming the full voltage developed in the armature at the time of the short circuit to be applied across a choke coil of reactance $(X_{ends} + X_{slots})$. The quantity X_{slots} can be obtained from formula

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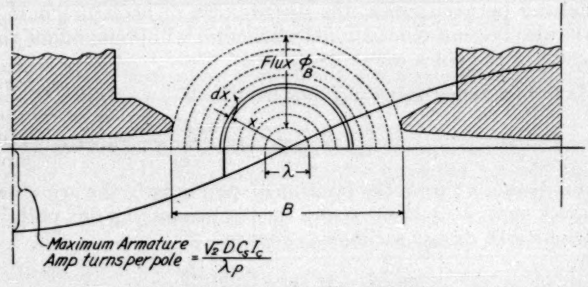
[4] There are a great many approximate formulas for the reactance of the overhanging portions of the winding (i.e. the portion which is outside the slots). A number of these will be found in recent contributions to the electrical engineering journals. The following are selected from the many books on electrical design: C. C. Hawkins, *The Dynamo*, sixth edition, Vol. III, page 200; Miles Walker, *Specification and Design of Dynamo Electric Machinery*, pages 388 and 426; Alfred Still, *Elements of Electrical Design*, page 339.

(17) by assuming the current I_c to be one ampere. This would be the correct value to use when all the phases are short-circuited. When only one or two phase-windings of a polyphase machine are short-circuited, a correction must be made in evaluating the amount of the self-induced flux which links with the windings.

In salient-pole machines there is yet another component of the total reactance to consider; this is the reactance of the flux paths over the cylindrical surface of the armature in the air space between the pole tips. This has not previously been considered because, under normal operating conditions, the tendency of the belt of armature conductors between the pole tips to establish flux from tooth to tooth and from pole tip to pole tip, is accounted for in any proper scheme for dealing with armature reaction. However, when a sudden short-circuit occurs, this core-surface leakage flux[5] may be an important factor in determining the short-circuit current, and the reactance of the leakage paths over the armature surface between pole tips should be considered as a real reactance to be added to those of the slots and end connections.

Calculation of Core-surface Reactance in Salient-pole Machines.—In order to calculate the leakage flux Φ_B of Fig. 6, the following simplifying assumptions will be made: (1) The portion of the armature face between pole tips is a plane sur-

FIG. 6.


Flux ϕ_B
dx
x
λ
B
Maximum Armature Amp turns per pole $= \frac{\sqrt{2}\,D C_s I_c}{\lambda p}$

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[5] Sometimes referred to as zig-zag leakage and belt leakage, being flux due to currents in the armature windings which is not accounted for as slot flux or end flux.

face. (2) The position of the conductors carrying maximum current is half way between pole tips. This is not unreasonable seeing that the power factor is nearly zero when a short-circuit occurs. (3) The distribution of current in the conductors is sinusoidal over the armature periphery. This assumption is as previously made; it involves the idea of very narrow "belts" of conductors, or a large number of slots with as many phases as there are slots. (4) The armature has a smooth surface, the fact that this surface is actually broken up by a succession of teeth and slots being disregarded. (5) The flux paths in air are semi-circular arcs described from a center on the armature periphery midway between pole tips. (6) The reluctance of the iron core is negligible. The symbols used are:

D = Diameter of armature core (inches),
l_a = Gross length of armature core (inches),
p = Number of poles,
B = Distance between pole tips (inches),
λ = Tooth pitch (inches),
C_s = Number of conductors per slot,
n_s = Number of slots per pole per phase,
I_c = R.m.s. value of current in armature windings (amperes).

With a sinusoidal current distribution having its maximum midway between poles, the distribution of armature m.m.f. will also be sinusoidal and its maximum will occur under the center of the poles where its value is

$$\text{Maximum armature m.m.f. per pole} = M = 0.4\pi \frac{\sqrt{2} D C_s I_c}{\lambda p} \text{ gilberts.} \qquad (18)$$

At a distance x from the position of zero m.m.f., the armature m.m.f. per pole is $M \sin x$, and the permeance of a flux path in air of width dx is $2.54 l_a dx/\pi x$, whence

$$d\Phi_B = 2M \sin x \frac{2.54 l_a dx}{\pi x}.$$

The lower limit of integration is $\lambda/2$ because the flux for smaller values of x has already been included in the formula for slot leakage flux. The upper limit is $B/2$, but since both these quantities are in inches, they must be multiplied by p/D to express them in radians. Therefore,

$$\Phi_B = \frac{2 \times 2.54 l_a}{\pi} M \int_{p\lambda/2D}^{pB/2D} \frac{\sin x}{x}.$$

The average value of the angle x between the limits considered is small, and no appreciable error will be introduced by assuming that $\sin x$ is equal to x, whence

$$\Phi_B = \frac{2 \times 2.54 l_a}{\pi} M \left(\frac{p}{2D}\right)(B - \lambda).$$

Substituting the value of M given by formula (18), we get

$$\Phi_B = 1.44 C_s I_c l_a \left(\frac{B - \lambda}{\lambda}\right) \text{ maxwells.} \qquad (19)$$

The voltage component or IX drop due to the "cutting" of this flux by the armature conductors may be calculated in the same manner as when developing formula (16) for the slot flux e.m.f. component; thus, on the sine-wave assumption,

$$IX_{\text{core surface}} = 4.44 \Phi_B f(C_s n_s p) 10^{-8}.$$

Substituting Φ_B from formula (19) we get

$$IX_{\text{core surface}} = I_c 6.4 f C_s^2 n_s p l_a \left(\frac{B - \lambda}{\lambda}\right) 10^{-8} \text{ volts.} \qquad (20)$$

It is well to remember that this is *not* a component of the total voltage developed in the armature windings under normal operating conditions.

If, instead of a reactive voltage component, the expression for the reactance $X_{\text{core surface}}$ is needed in order to add this to X_{ends} and X_{slots}, and so obtain the total reactance (in ohms) which determines the amount of the initial short-circuit current, it is merely necessary to omit I_c from the formula (20).

Concluding Remarks.—It is possible that some of the assumptions made in this article may be subject to criticism. The object has been to simplify the problem and avoid the introduction of minor correcting factors which might introduce complications. Every designer uses formulas which involve many simplifying assumptions and also constants based upon his experience with the particular type of machine with which he is concerned. The writer's aim has been to simplify and elucidate a difficult subject by avoiding refinements of calculation and by consistently representing the voltage components in the armature windings as being due to the "cutting" of the flux components by conductors which are moving relatively to the magnetic fields, the latter being considered fixed in space if the armature is rotating, or rotating with the field system if the armature is stationary.

Many of the devices used by designers are artificial, or based on mathematical rather than physical conceptions. These yield excellent results when used by those familiar with their application, but are rarely of much value in designs embodying radical departures from standard designs. It is then advisable to form a clear mental picture of the various paths taken by the magnetic flux and consider where and how every portion of this flux does or does not link with the armature windings. The convenient and valuable, but rather artificial distinction between direct and transverse reactions (and reactances) based on Prof. A. Blondel's two-reaction method of determining regulation, is very useful when the proper constants are available for use in the formulas, but these constants are not available in connection with any but the more or less standardized types of machines.

One of the assumptions here made is that of full-pitch windings which are admittedly unusual in modern machines using double-layer windings and a coil pitch of about two-thirds. It is, however, not a difficult matter to determine what the value of the ampere-conductors in each slot will be when the currents in the upper and lower coil-sides are not equal as here assumed. The proper correction to the quantity $(C_s i_s)$ in the formulas can then be made. Another assumption is that of very narrow "belts" of conductors. This, again, causes the current distribution in the successive slots to be different from the actual distribution; but the writer believes this simplification to be justified when the number of phases is three or more. The assumption that the path of the slot leakage flux is the shortest straight-line distance across the slot, although incorrect, is not unusual. The error introduced by this assumption is generally considered to be negligible.

COMPARISON BETWEEN ARTIFICIAL AND NATURAL SOLARIZATION AND STABILIZATION OF SPECIAL ULTRAVIOLET TRANSMITTING GLASSES COMMERCIALLY ON THE MARKET.

BY

DONALD C. STOCKBARGER, Sc.D., and LYMAN B. JOHNSON, S.M.

As reported by Stockbarger, Dingee and Burns,[1] artificial solarization of glass follows a course dependent on the quality of the radiation. Glass exposed to bare mercury arc radiation was found to depreciate further than similar glass which was protected by filters from the short wave-length radiation. Specifically, natural solarization from exposure to sunlight was found to be less severe than bare arc solarization, but to be similar to that resulting from exposure to filtered arc radiation.

It was evident that the only perfectly fair way to conduct solarization tests was to adopt a kind of radiation which would have as nearly as possible the same effect on glass as would sunlight. To that end a comparison of filters was made until one was found which seemed to be satisfactory. This was a thin sheet of neutral glass whose transmission terminated at about 3000 A. Although it has been employed continuously for many months, no thorough test of its merits has been made until recently.

This paper reports the results of a large number of tests made to compare the solarization produced by bare quartz mercury arc, filtered mercury arc and solar radiation. In order that these tests might have weight they were made on several brands of glass. Another purpose of the tests being to make a survey of the open market to learn something concerning the average quality of the glass sold to the public, most of the test samples were cut from sheets purchased for us by uninterested persons from centers widely scattered over the United States and from England.

Inasmuch as interest usually centers around the transmission at the wave-length 3025 A. and in order to reduce the amount of routine work the majority of the transmission measurements were made at this point only. The measuring instrument was a spectroradiometer equipped with a thermopile.

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[1] Stockbarger, Dingee and Burns, *Modern Hospital*, 31, p. 144, 1928.

Solarization by bare and filtered quartz mercury arc radiation was accomplished by exposing samples for 200 hours at a distance of 18 inches from the arc. In the case of the filtered arc solarization the filter was placed close to the samples and precautions were taken to prevent unfiltered radiation from reaching the glass. The source was a six-inch vertical Uviarc operated at 4 amperes and 170 volts across the arc.

Natural solarization was carried out on the roof of a building during the months of January, February and March of this year, except in the cases of samples 43–47 inclusive which were exposed to autumn sunlight for approximately the same length of time. None of the samples so treated had previously been exposed to ultraviolet radiation.

Wood and Leathwood [2] reported that glass which had been oversolarized by exposure to bare quartz mercury arc radiation could be rejuvenated partially by subsequent exposure to sunlight and that the final transmissions were found to be the same as though the glass had been exposed to sunlight only. To verify their findings and to determine whether or not the return to normal solarization condition would take place within the time required for natural solarization, another set of samples cut from sheets 1–42 inclusive was placed on the roof after receiving bare quartz mercury arc radiation for 200 hours.

Because of the variation in thickness of glass with which we had to work it was necessary to reduce all transmission results to equivalent values for a standard thickness (two millimeters) by use of the well-known formula

$$\log \frac{T'}{0.92} = \frac{b'}{b} \log \frac{T}{0.92},$$

where T = transmission for thickness b
and T' = transmission for thickness b'.

The results of the transmission measurements are summarized in Table 1 and are self-explanatory. Comparing

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[2] Wood and Leathwood, *Nature*, London, **124**, p. 441, 1929.

TABLE I.

Values of Transmission † at 3025 A. before and after Solarization.

Sample No.	Kind of Glass.	Origin.	Per cent. Transmission.				
			Original.	Filtered Arc.	Sun.	Bare Arc.	Bare Arc and Sun.
1	B	Mass.	64	52	52	35	—
2	B	Conn.	63	49	47	30	—
3	B	Ohio	54	37	38	25	—
4	B	Conn.	64	48	48	30	—
5	B	Ohio	64	49	—	33	—
6	B	R. I.	67	56	54	39	—
7	C	Ill.	65	60	60	47	—
8	B	Ill.	61	48	46	32	41
9	C	Ohio	45	30	31	21	—
10	C	Ohio	38	33	33	22	—
11	B	Mass.	61	48	49	23	42
12	A	N. Y.	72	59	58	48	—
13	A	N. Y.	75	58	58	40	—
14	C	Wis.	48	36	35	25	—
15	A	N. Y.	75	62	62	47	58
16	B	N. Y.	51	37	36	24	—
17	C	N. Y.	56	47	47	33	—
18	C	Texas	67	64	63	49	—
19	B	D. C.	59	50	48	30	—
20	C	Colo.	62	56	56	44	—
21	B	N. Y.	56	47	45	31	—
22	C	N. Y.	65	53	53	42	49
23	B	Colo.	66	54	53	37	—
24	C	N. Y.	64	55	56	46	—
25	B	Eng.	62	51	50	34	—
26	B	Eng.	62	49	49	32	—
27	B	Eng.	57	47	48	32	44
28	B	Calif.	63	51	50	31	—
29	B	N. Y.	63	49	47	32	—
30	C	N. Y.	64	56	56	44	52
31	C	N. J.	68	65	64	51	—
32	C	N. Y.	60	56	58	45	48
33	B	N. Y.	63	50	48	32	—
34	B	N. J.	64	48	49	32	—
35	B	N. J.	62	47	46	25	—
36	B	N. Y.	63	49	48	31	—
37	B	N. J.	58	47	46	32	—
38	B	N. Y.	62	48	45	30	—
39	B	N. Y.	58	47	46	29	—
40	B	N. Y.	64	51	49	32	—
41	A	N. Y.	74	62	62	44	—
42	B	Pa.	53	40	38	24	—
43	B	*—	62	48	47	32	—
44	B	*—	63	48	47	31	—
45	C	*—	43	34	34	27	—
46	C	*—	54	42	41	30	—
47	C	*—	40	35	35	26	—

---

* Supplied with compliments of local sales offices.

† For 2 mm. thickness.

filtered arc solarization with that produced by sunlight it is seen that in some cases the former was a little more severe while in others it was a little less severe. It is evident that the bare arc solarization was much too severe in every case and therefore it is difficult to see what this method has to recommend it. The filtered arc method is to be preferred.

The close agreement between filtered arc and sunlight results is taken as proof that our filtered arc radiation is a satisfactory substitute for sunlight in work of this kind. Answering a question which might well be raised, we have found that it does not matter whether a quartz mercury arc or a flaming carbon arc be used with the filter. In a number of comparative tests made a year ago the solarization results were the same in every case within the limit of experimental error.

A limited number of samples were tested after exposure to both bare arc radiation and sunlight because it was found that the rejuvenation process had not reached completion. From this fact we would conclude that during the winter months, at any rate, the rejuvenation proceeds somewhat more slowly than does solarization.

TABLE 2.

*Average Values of Transmission * at 3025 A. before and after Solarization.*

Kind of glass.†	Per cent. Transmission.			
	Original.	Filtered Arc.	Sun.	Bare Arc.
B............	61	48	47	31
C............	56	48	48	37

* For 2 mm. thickness.

† Glass A has not been included because of the small number of samples which were all obtained from one dealer.

Referring to Table 2, three conclusions are drawn. Glass B was somewhat higher in average original transmission than glass C of equal thickness, but after filtered arc solarization the two had equal transmissions. Natural solarization produced substantially the same effect as did filtered arc solarization. Bare arc solarization was more severe for glass B than for glass C.

Transmission measurements made at intervals during the roof exposures indicated that the natural solarization had reached completion at the end of the exposure period. Furthermore the same agreement between filtered arc and natural solarization has been found in two additional tests in which the natural solarization process extended over periods of one and one-half years and two and one-half years. It appears safe to conclude therefore that the bare arc solarization tests were unfair not only because they yielded results which were too low but also because they indicated a difference between the qualities of B and C which was meaningless. Since in practice windows are exposed to sunlight and not to bare arcs, tests should be made by means of filtered arc radiation when sunlight cannot be used.

The fact that the special ultraviolet transmitting glasses become stable after a period of seasoning is not new. It has been proved many times during the past few years that after solarization their high transmissions, for rays to which ordinary window materials are opaque, are permanent properties. The tests described in this paper furnish further proof.

RADIATION MEASUREMENTS LABORATORY,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
CAMBRIDGE, MASSACHUSETTS,
April 28, 1930.

Prevention of Rancidity. GEORGE W. FIERO (*Am. J. Pharmacy*, 1930, **102**, 146–154) has studied the influence of 30 different preservatives, either chemical compounds or essential oils, on the development of rancidity in pure fresh leaf lard which was kept in unstoppered test tubes. Rancidity was recognized by its characteristic odor and taste and, whenever possible, by the Kreis reaction (development of a pink or red color on shaking 5 cc. of molten fat with 5 cc. of concentrated hydrochloric acid and adding 5 cc. of 0.1 per cent. solution of phloroglucinol in ether).

The lard was preserved without development of rancidity for 8 weeks in a warm light room or 25 weeks in a cool dark room by (1) resorcinol 1 per cent., (2) oil of clove 0.38 per cent., (3) guaiacol 0.06 per cent., (4) thymol 2.5 per cent., (5) creosote 0.25 per cent.

It was preserved for 4 weeks in a warm light room or 10 weeks in a cool dark room, but not for the longer periods of storage, by (1) safrol 0.5 per cent., (2) sodium benzoate 1 per cent., (3) sodium benzoate (dissolved in water and emulsified with the lard by means of neutral soap) 0.1 per cent.

Methenamine prevented the development of rancidity; but the lard became yellow on standing. Rancidity did not develop in mixtures of lard with (1) hydrogenated oil 25 per cent., (2) white wax and petrolatum 25 per cent. of each, (3) white wax and liquid petrolatum 25 per cent. of each, (4) paraffin and liquid petrolatum 25 per cent. of each, or in the following ointment bases: White wax, yellow wax, paraffin, wool fat, hydrous wool fat, white petrolatum, yellow petrolatum, liquid petrolatum, hydrogenated oil, or their mixtures.

J. S. H.

RADIOACTIVE PROPERTIES OF ROCKS, SOILS, CRUDE OIL AND WATERS FROM SOUTHERN CALIFORNIA.

BY

J. LLOYD BOHN, B.S., Ph.D.,

California Institute of Technology, Pasadena.

ABSTRACT.

This paper includes measurements on waters from Lake Arrowhead, Arrowhead Hot Springs, Harlem Hot Springs, the Pacific Ocean, and from numerous wells and tunnels of the Pasadena and neighboring water supplies. It also includes measurements on rocks, soils, and crude oil.

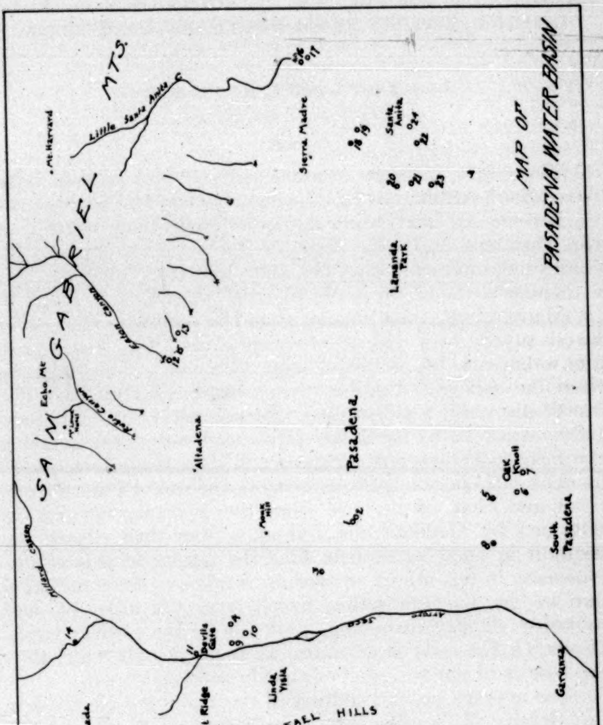
The activity of the water of Lake Arrowhead is found to be one hundred times smaller than the minimum that could be detected with the electroscopes used for cosmic ray measurements by Millikan and Cameron. This lake is, therefore, very suitable for cosmic ray measurements. The Pasadena wells yield waters approximately one thousand times as active as the Lake Arrowhead water, thus making the Pasadena reservoirs unsuitable for these measurements.

The ratio between the activity of the soil at Lake Arrowhead and that of the soil from the Institute campus, as obtained by Millikan and Cameron from their cosmic ray data, is in good agreement with the results in this report. However, in this report no account is taken of the γ-radiation due to the thorium series, which is of the order of, and probably slightly exceeding, that due to the radium series. If the Th/Ra ratio is constant, as is usually the case, then the results of the two methods are in good agreement.

The average radium content of the acid intrusive rocks in the vicinity of Devil's Gate Dam, Pasadena, is found to be 2.3×10^{-12} gms. of radium per gm. of rock.

Pasadena tap water contains about 1 eman or 1×10^{-10} curies per liter, while the average of the sources is about five times as great. The radium emanation found in crude oil was of the same order of magnitude as that found in well water.

Carnotite when fused with carbonates at about 1000° C.


MAP OF
PASADENA WATER BASIN
SAN GABRIEL MTS.
Mt. Harvard
Little Santa Anita C.
Eaton Canyon
Echo Mt.
Rubio Canyon
Millard Canyon
Sierra Madre
Santa Anita
Lamanda Park
Altadena
Pasadena
Monk Hill
Oak Knoll
South Pasadena
Garvanza
Arroyo Seco
Devils Gate
Flint Ridge
Linda Vista
La Canada
SAN RAFAEL HILLS
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9
10
11
12
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loses part of its Ra—probably as a sublimate of $RaBr_2$. Such a loss in fusing rocks might account for the discrepancy between the solution and fusion methods as found by J. Joly.

INTRODUCTION.

Many extensive investigations on the distribution of radioactive elements throughout the crust of the earth have been carried out by Strutt,[1] Joly,[2] Elster and Geitel[3] and many others, but so far the Southern California Region has been almost, if not entirely, devoid of such investigation. It is, therefore, quite essential to have data on this region that may be available for the various problems dealing with geology.

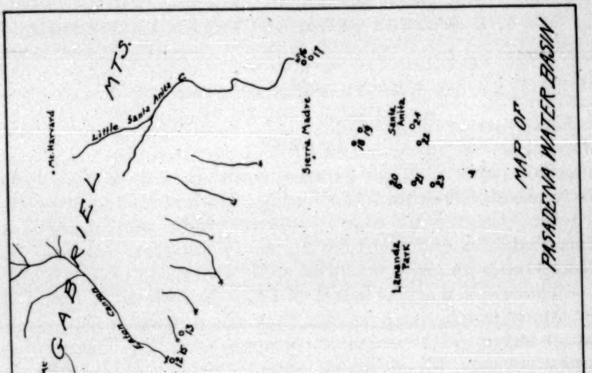
However, the chief reason for starting this investigation was that of finding the activity of the waters and the soils connected with the cosmic ray measurements of Millikan and Cameron.[4]

Many of the data included in this report relate to the water supply of the City of Pasadena. Numerous tunnels and wells, as well as rocks from the vicinity of the tunnels and from the drill cores of the wells have been measured. The report also includes data on waters from the Pacific Ocean, Lake Arrowhead, Arrowhead Hot Springs, Harlem Hot Springs, and numerous sources east of Pasadena. Measurements on crude oil and on soils are included.

Some of the problems of geological interest are: (1) the relation between the activity of waters and the underground structure of the rocks through which these waters flow—especially the relation to faults. (2) The distribution of radioactivity in the rocks of the lithosphere and the relation to origin, composition and depth of the rocks studied. Directly related to these is the question of the influence of the heat developed by radioactive materials upon the surface history of the earth which has not yet been satisfactorily explained. These problems are being investigated at present and will be the object of a separate paper by Dr. Engel of the Department of Geology.

APPARATUS.

The furnace is almost identical with that used by Joly.[5] Fig. 1 represents a cross-section of the furnace. The base, of


MAP OF
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Pasadena
Oak Knoll
South Pasadena
Garvanza
Devils Gate
Flint Ridge
Linda Vista
La Canada
SAN RAFAEL HILLS
Arroyo Seco
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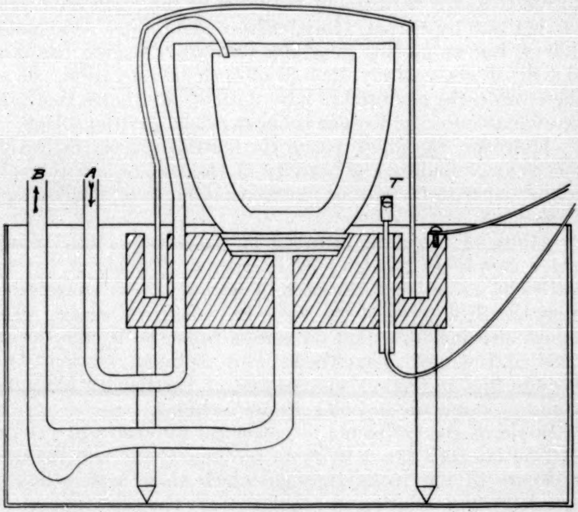
APPARATUS.

The furnace is almost identical with that used by Joly.[5] Fig. 1 represents a cross-section of the furnace. The base, of

Swedish iron, is four inches square and about one and one-eighth inches thick and has four supporting legs of brass. An annular groove about three-fourths of an inch deep is cut into it for admitting the pyrex cover. When in operation the groove is filled with mercury and the apparatus set into

FIG. 1.



a pyrex crystallizing dish which is completely filled with water, thus covering the base and the mercury outside of the pyrex cover. The purpose of the water is to keep the base and the mercury seal cool when the furnace is hot. Under the pyrex cover we have a quartz crucible of about 150 c.c. capacity resting upon several layers of asbestos for thermal insulation. Inside this quartz crucible is the heating element, which is not shown in the diagram, but which is put into position similar to that used by Joly, i.e., a sort of basket arrangement with vertical loops. One end of this heating element, a platinum wire 1.2 mm. in diameter and slightly more than a meter in length, is connected to the base and the other end to the insulated terminal (*T*) shown in the diagram under the pyrex cover. An old spark plug with mica insulation was found to be very suitable for this terminal. Inside, a platinum crucible of 60 c.c. capacity is imbedded in magnesium oxide which serves several purposes; namely that of electrically insulating the platinum crucible from the heating element and of thermally insulating the heating element from the outside. The heating element is placed as close to the platinum crucible as possible without danger of actual contact. The platinum crucible has a cover with two openings in it; one for observing the mixture while fusing, and to obtain the temperature with an optical pyrometer, and the other for admitting air through inlet (*A*) to scavenge the furnace and thus remove the residual emanation. The outlet (*B*) for the emanation, air, carbon dioxide, water vapor, etc., upon fusion is through the tube in the center of the base. This leads to the absorbing tubes of soda lime. The bulge in the glass tubing at the bottom serves as a reservoir for the condensed water vapor.

The apparatus used for separating emanation from solutions is of the ordinary Boltwood type and needs no further description.

Two electroscopes were used for determining the radioactive content of waters and thus one served as a check on the other when samples from the same source were tested in both electroscopes. Electroscope designated as No. 1 is of the ordinary gold leaf type mounted on top of an emanation chamber of about 600 c.c. capacity. Electroscope No. 2 is of the C. T. R. Wilson type and was used for the solid materials as well as for water. In this instrument both emanation chamber and electroscope are combined in one, thus decreasing the electrical capacity and thereby increasing the sensitivity. A gold leaf of 1½ mm. width is mounted on a flat strip of polished metal, which is suspended from the top of the electroscope by a small quartz rod. On account of the hygroscopic nature of quartz the admitted gases must be free from water vapor. This is accomplished by passing all air and emanation, after the CO_2 has been removed by the soda lime, through a trap of concentrated H_2SO_4 and then through a small tube of P_2O_5 before entering the electroscope.

CALIBRATION.

Carnotite of known uranium content was used for the purpose of calibration. For the calibration to be used with the solution method for rocks and the water analysis a carefully weighed quantity of carnotite was fused with sodium and potassium carbonates and then put into solution. This solution was put into a flask and boiled to remove all emanation and was then securely corked. After a number of days the solution was boiled and the evolved gases passed through the Boltwood apparatus filled with hot NaOH solution and through drying tubes into the electroscope.

A sample calculation is here given:

.0107 gm. of carnotite of 1.58 per cent. uranium content were used.
2 days $17\frac{2}{3}$ hours = time stored.
.391 = part of emanation accumulated.
3.40×10^{-7} gms. of Ra in equilibrium with 1 gm. of uranium.[6]
Leak of electroscope = 4.0 divisions per hour.
Emanation entered electroscope at 8:15 P.M.
Initial reading = 1.4 at 8:45.
Final reading = 34.4 at 9:45.
Leak due to emanation = 33.0 − 4.0 = 29.0 divisions per hour.

$$\frac{.0107 \times .0158 \times .391 \times 3.40 \times 10^{-7}}{29.0} = .776 \times 10^{-12} \text{ gms. Radium per scale division per hour} = \text{calibration constant.}$$

The average constant for electroscope No. 2 as used for water determinations was .780 gm. Ra per div. per hour.

In order to make a check on this value Mr. Ransom of this laboratory kindly allowed us to use a standard radium solution he had in his possession. With this solution we obtained a value for the constant of .775 gm. Ra per div. We, therefore, feel quite certain that the original calibration was correct.

To calibrate by the fusion method a quantity of the standard carnotite was mixed with about four grams of rock, which had been tested previously and for which the electroscope leak per hour per gram of rock was known. The reason for adding the rock was to obtain greater effervescence and to meet as nearly as possible the actual conditions of making determinations on rocks. The result was then checked with a standard solution.

It was found that the furnace became contaminated with radium during calibration and had to be cleaned with acid. The contamination no doubt was caused by the comparatively low sublimation point of some compound of radium—probably $RaBr_2$.

DATA RELATED TO COSMIC RAY MEASUREMENTS.

Water was taken from Lake Arrowhead at various depths down to 50 feet to see whether there was any variation of activity with depth, but within the accuracy of the measurements no such variation was detected. The average result was found to be $.58 \times 10^{-12}$ curies per liter or 5.8×10^{-16} curies per c.c. Data relating to the Pasadena water supply will be found further on in this report, but for our present purpose it is sufficient to know that the average activity was found to be approximately 1000 times greater than that of Lake Arrowhead.

The ionization per c.c. due to γ-rays from RaC in equilibrium with one gram of Ra and at a distance r is given by the equation:

$$i = \frac{K \cdot e^{-ur}}{r^2},$$

where K, Eve's constant, is the number of pairs of ions produced per c.c. per sec. by the γ-rays of RaC in equilibrium with one gram of Ra when at a distance of 1 cm. in air; u is the absorption coefficient for γ-rays. For the ionization inside an electroscope of radius R, due to an outside Ra concentration, p, we get the integral:

$$\begin{aligned} I &= \int_R^\infty 4\pi r^2 pKe^{-ur}dr/r^2 \\ &= \frac{4\pi pKe^{-uR}}{u}. \end{aligned}$$

Substituting the following values:

$R = 7.5$ cm.,
$K = 5.5 \times 10^9$ ion pairs per sec. per gm. Ra per cm.,
$u = .055$ cm.$^{-1}$ for water,
$p = 5.8 \times 10^{-16}$ gms. Ra per c.c.,
we get $I = .00048$ ion pairs per c.c. per sec.

It was found by Millikan and Cameron (results not yet published) that 100 times the above value would just be on the border line of causing trouble while 1000 times would create a considerable error in cosmic ray data. It was for this reason that, as a result of preliminary measurements, they abandoned the Pasadena reservoirs so far as cosmic ray measurements were concerned and chose the more suitable waters of Arrowhead, Muir and Gem lakes.

The following are the values obtained for the activity of the soils in the vicinity of cosmic ray measurements.

Lake Arrowhead.	Institute Campus.
$4.07 \times 10^{-12} \frac{\text{gm. Ra}}{\text{gm. soil}}$	$2.30 \times 10^{-12} \frac{\text{gm. Ra}}{\text{gm. soil}}$
4.01	2.04
3.92	2.42
3.70	2.32
3.81	2.62
3.97	2.28
Average $3.90 \times 10^{-12} \frac{\text{gm. Ra}}{\text{gm. soil}}$	$2.33 \times 10^{-12} \frac{\text{gm. Ra}}{\text{gm. soil}}$

This gives a ratio of 1.67 between the activities of the two localities named.

Millikan and Cameron (not published) found with different electroscopes with different internal pressures and at different times the following ratios: 1.44, 1.60 and 1.53—an average of 1.52. The agreement between the results obtained by the widely different methods is fairly close, but they should be taken as qualitative only. Millikan and Cameron measured not only the γ-radiations of the Ra series but also those of the Th series. But since the Th/U ratio for ordinary rocks and soils varies very little and has an average value of 2.3 for acid rocks; [7] and since the γ-ray equivalent of one gm. of Ra is 6.85×10^6 gms. Th [8] the γ-radiation due to the Th series would be just equal to that of the Ra series. However, the former probably produces more ionization due to the harder radiation and therefore greater effective distance. If the Th/U ratio varied between the two localities the ratio between the total γ-radiations and the γ-radiations due to Ra should vary only by an amount half as great.

DATA ON WATERS.

Pasadena Water Supply.

Numbers on Map.	Source.	Electroscope No. 1.	Electroscope No. 2.	Average.
	I. WELLS.			
1	Garfield	9.2×10^{-10}	9.1×10^{-10}	9.2×10^{-10} curies/liter
2	Villa	8.4	8.3	
			7.9	8.2
3	Sunset		6.4	6.4
4	Sheldon	4.8		4.8
5	Oakland at Glenarm	3.5	3.6	3.6
6	Los Robles	7.5	7.1	7.3
7	Huntington	0.10		0.10
8	Busch	3.7	3.7	3.7
	II. TUNNELS.			
9	Richardson	5.6×10^{-10}	5.5×10^{-10}	5.6×10^{-10} curies/liter
10	Wilson		5.5	5.5
11	Devils Gate	5.0	5.6	5.3
12	Eaton No. 3	3.6	3.8	3.7
13	Post	2.1		
		1.6		1.9
14	Arroyo Seco Spring	0.11	0.13	0.12
15	Eaton Div.	0.78		0.78

III. PASADENA TAP WATER.

Lowest value	0.25×10^{-10} curies/liter
Highest value	1.90
Average of 20 determinations	0.97

Sierra Madre.

Numbers on Map.	Source.	Electroscope No. 1.	Electroscope No. 2.	Average.
16	No. 1	2.8×10^{-10}	3.0×10^{-10}	2.9×10^{-10} curies/liter
17	No. 2		3.2	3.2

Anita Baldwin Estate.

18	No. 1	3.0		3.0
19	No. 2	3.5	3.2	3.3

Baldwin Rancho.

20	No. 1		3.6	3.6
21	No. 2	6.3		6.3
22	No. 5	5.2		5.2
23	No. 6		7.1	7.1
24	No. 7	3.3		3.3

Source.	Electroscope No. 1.	Electroscope No. 2.	Average.
Arrowhead Hot Springs			
Penyugal	2.3	2.1	2.2
Granite	1.9	2.1	2.0
Palm		3.7	3.7
Harlem Hot Springs	18.0		18.0
Sea Water at Venice Pier		0.77×10^{-12}	
		1.10×10^{-12}	0.94×10^{-12}
Lake Arrowhead	Surface	0.63	
	−40 ft.	.49 and .51	0.54×10^{-12}

DATA ON ROCKS.

ROCKS FROM THE VICINITY OF DEVIL'S GATE DAM, PASADENA.

1. Granodiorite: 2.27×10^{-12} gm. Ra/gm. rock. $3.02 \times 10^{-12} \cdot 2.73 \times 10^{-12}$, 2.48×10^{-12}, Average: 2.6×10^{-12} gm. Ra/gm. rock.
2. Aplite: 1.67, 2.32, 2.02, Average: 2.0×10^{-12} gm. Ra/gm. rock.
3. Quartz Diorite Gneis: 2.46, 2.30, 2.13, 2.44, Average: 2.3×10^{-12}.
4. Granodiorite (from Arroyo Seco): 2.76, 2.46, 2.60, Average: 2.6×10^{-12}.

DRILL CUTTINGS FROM PASADENA WATER WELLS.

Wells.	Depth.	Rock.	$\times 10^{-12}$.	Gm. Ra/Gm. Rock.	Average.
Ventura	485 ft.	Granodiorite	1.45	1.40	1.4
Sunset	750 ft.	"	0.77	—	0.8
Sheldon	190 ft.	Aplite	1.00	0.70	0.9
Villa	950 ft.	Quartz diorite gneiss	1.21	1.29	1.3

SOILS.

Campus California Institute of Technology	2.3×10^{-12} gm. Ra/gm. soil
Near Arrowhead Lake	3.9 " " "

CRUDE OIL FROM MONTEBELLO OIL FIELD.

			Average.
Well—Temple No. 15	1.9×10^{-10}	1.8×10^{-10}	1.9×10^{-10} curies/l.
Well—Temple No. 9	2.0 "	2.3 "	2.2 " "

DISCUSSION.

With a few exceptions, the results obtained on the waters from all the wells, tunnels and springs are included between the values of from 2 to 9×10^{-10} curies per liter. This would tend to indicate that all these waters come from rock formations having approximately the same radium content. The determinations made on the rocks of the Pasadena water basin confirm this point of view since the results are included between 2.0 and 2.6×10^{-12} grams of radium per gram of rock. The lower values obtained on drill cuttings indicate a loss of radium probably due to solution in water during the churn-drilling process.

Among the waters, the lowest values correspond to the sources where the proportion of surface waters is the largest. Such is the case of Arroyo Seco spring, the shallow turbulent stream of Eaton diversion, and very likely of the Huntington well.

The activity of Pasadena tap water is found to vary considerably and on the average has an activity of only about one-fifth of the average activity of the sources. The loss of emanation is mainly due to the storage of water in reservoirs.

Harlem Hot spring yields water at 90° F. and with an activity of 18×10^{-10} curies per liter. This is twice the maximum activity found in Pasadena. It is reported that following the San Jacinto earthquake of 1918, the temperature dropped slowly from 120° F. to the present temperature of 90° F. It is thus probable that this water, or at least part of it, has a deep seated origin along a fault, in the same way as the Arrowhead Hot Springs (160° to 200° F.) issue along the faulted zone of the San Andreas rift. The low activity of these last waters is very likely due to their high temperature which favors the escape of the emanation with the gases issuing from the springs.

The samples of crude oil had an average emanation content of 2.0×10^{-10} curies per liter thus of the same order as that found in waters. However, due to the unsatisfactory method of collecting samples, some of the emanation escaped and this value must be considered as being slightly below the real one. Samples of oil were stored for one month and then tested for radium with negative results. This confirms the conclusion previously reached by Engler [9] that the radioactivity of crude oil is due to emanation alone and not to the presence of radium salts in solution.

The loss of Ra by sublimation upon fusing with carbonates at high temperature might easily cause the discrepancy between the solution and fusion methods as found by Joly. In the fusion method all the emanation present would still be collected, but when putting the fused material into solution only part of the Ra would be present thus giving lower values by the solution method.

In conclusion, the writer wishes to express his thanks to the Pasadena Water Department, particularly to Mr. S. B. Morris, Superintendent and Chief Engineer, and Mr. C. W. Sopp, Assistant Engineer, for their coöperation and help in obtaining samples of water. He is also indebted to various members of the California Institute of Technology for their suggestions and advice, especially to Dr. Millikan and Dr. Cameron, for permitting the use of unpublished data, to Mr. Ransom for furnishing a standard radium solution used in calibration and to Dr. Engel for determining the rock specimens and for his coöperation in the course of this investigation.

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THE MECHANISM OF SPARK DISCHARGES.

BY

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In his paper of the above title,[1] Professor Loeb criticizes rather severely my suggestion[2] that during breakdown of spark gaps at atmospheric pressure, the electrical energy input density in the path of an advancing electron which is multiplying by collision becomes so great that the temperature of the gas rises to a value high enough for thermal ionisation to set in. Professor Loeb points out rightly that according to present day views the collisions of electrons with molecules are elastic except when excitation or ionisation is produced, so that only a very small part, a ten thousandth or less, of the energy input into the electron swarm goes to directly increasing the translational kinetic energy of the gas molecules. Hence, if the temperature of the gas (which is not in an equilibrium condition) is taken to be that corresponding to the mean kinetic energy of the molecules, Professor Loeb concludes that my calculated temperatures are ten thousand or more times too large.

However, what does a mere factor of ten thousand or more amount to, when there is any uncertainty in the value of the ionising constant α which enters into the formulæ in the exponential factor $e^{\alpha d}$? At $d = 1$ cm., α only needs to be increased from 13, the value which I used, to $\alpha = 22.2$ to multiply the energy input density by 10^4. If α is taken as 40, a value which Professor Loeb himself has used, the energy input density is multiplied by more than 10^{11}.

My *Electrical World* paper was one of the first to offer two important suggestions as to how the Townsend theory of spark over might be modified so as to account for the observed high speed of breakdown of spark gaps at atmospheric pressures. The first was that early in the breakdown process, in fact during the passage across the gap of the first electron, space charges would develop which would greatly modify the electrical field, so that a large increase in $\bar{\alpha}$ for some of the electrons would result. This idea seems to be generally accepted by investigators in this field.

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[1] J. OF F. INST., 210, 15, 1930.

[2] *Eectrical World*, 91, 761, 1928.

The second suggestion, and one which I also think is important to account for the rapid development of heavy current discharges as observed by electrical engineers, was the entering into the breakdown process of some other ionising agent, one which did not require the continued existence of an intense electrical field necessary for ionisation by collision in the ordinary sense. This ionising agent I believe is closely bound up with the high energy density input which develops during the breakdown process and may I think be described as thermal ionisation.

The electrical engineer observes the breakdown of gaps with current rising to thousands of amperes in fractions of a microsecond. Ionisation by collision, together with the distortion of the electric field by space charges alone cannot explain these discharges. This is because an increase of the electric field at one point by distortion with a large increase in α, means necessarily a decrease in the field and a large decrease in α at other points. At these other points then there is relatively no production of ions. Now, however profuse the production of electrons and ions may be in the strong parts of the field, the current which can flow through the relatively ion-free weak parts of the field is necessarily limited to a small value by the space charges accompanying such flow of current.

It may be that at low pressures, where ion and electron mobilities are larger, and for circuits in which the power is very limited, breakdown may involve currents so small that they may be carried by weak fields across portions of the gap in which the ionising activity is small. For example Rogowski and Tamm,[3] show pictures of the course of breakdown of a spark gap which resemble glows and suggest only ionisation by collision. However, these experiments were made at pressures of 50 mm. and less and the current was only two or three amperes. Such a current would not constitute a breakdown for the circuit of the electrical engineer. The surge impedance of his supply circuit being only a few hundred ohms, such a current would cause his applied voltage to drop by only a kilovolt or so. Furthermore, at atmospheric pressures a large current glow is very unstable, and the discharge tends to concentrate into a form resembling more the positive column of a large current arc. Even in Rogowski and Tamm's experiments just mentioned, the diffuse glow form of discharge began to develop a concentrated streamer after a few microseconds. My opinion is that at atmospheric pressure, and with large currents the concentrated streamer form of discharge forms before the glow type, and prevents the glow form from appearing at all. This is confirmed by the observations of Torok who finds that intense streamers form in fractions of a microsecond. Measurements of the current simultaneous with the observation of the streamers reveal thousands of amperes.[4] The current density in the streamers is undoubtedly thousands of amperes per cm^2. My opinion is that ionisation by collision in the ordinary sense, alone cannot build up such large conductivity in such short times, particularly, when it is remembered that in the streamers while the current is growing, the electric gradient is very much reduced. I think it is not unreasonable to ascribe the growth of conductivity in these streamers to the same agents which are responsible for the production of ions in the positive column of the heavy current arc. There is little doubt that these ionising agents are closely related to the high energy input, and Professor K. T. Compton[5] has not hesitated to describe it as thermal ionisation.

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[3] Rogowski and Tamm, *Arch. f. Elektrot.*, **XX**, 625, 1928.

The theory which I give in my *Electrical World* paper makes the thermal ionisation in a streamer play an important part in accelerating the breakdown of a spark. It must not be overlooked however, that my theory also calls for the development of space charges with increase of field and α, antecedent to the development of thermal ionisation. With the field acting on the advance electrons rising to two or three times 3.10^4 volts per cm., α goes up to values of several hundred, and the remarks of Professor Loeb concerning the inefficiency of transfer of electron energy to molecular translational energy, while probably true, become pointless.

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[4] Slepian and Torok, *Eect. J.*, **XXVI**, 107, 1929.

[5] K. T. Compton, *Phys. Rev.*, 21, 287, 1923.

Influence of Geographical Location upon Basal Metabolism. The basal metabolism is the rate of production of heat by an individual while in the post-absorptive condition and in a state of rest and relaxation. It is the energy required to maintain the body temperature, circulation, respiration, the work of the ductless glands, and necessary cellular function in the resting animal. It varies with the height, weight, age, and sex. Recent work of Jennie Tilt (*J. Biol. Chem.*, 1930, **86**, 635–641) indicates that basal metabolism may also be influenced by geographical location. She used as subjects 52 normal young women, college students in Florida, who either were natives of that state or else had lived there most of their lives. The basal metabolism of these women tended to be lower than that predicted for women of the same age living in the north. Repeated determinations, made on certain members of this group at intervals of several months, showed that their basal metabolism was not influenced by the season of the year.

J. S. H.

New Test for Irish Moss. According to GEORGE E. EWE (*J. Am. Pharmaceutical Asso.*, 1930, **19**, 568–570) a solution of methylene blue yields a deep blue precipitate with the decoction of Irish moss. The precipitate forms in either the hot or the cold decoction, and is not produced by decoctions of tragacanth, Indian gum, and acacia, or by gelatin. When the other gums are mixed with Irish moss, the precipitate forms, but is more finely divided.

J. S. H.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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ANALYSIS OF THE REACTIONS, FIG. 67.

For the kinetic equilibrium of the upper connecting rod, the inertia components are

$$K' = m_l'\left(\frac{k^2 - l_g'l'}{l'}\right)\ddot{\psi} = \left(\frac{I_l'}{l'} - m_l'l_g'\right)\ddot{\psi},$$

$$X_c' = m_l'\left(\frac{l' - l_g'}{l'}\right)\ddot{x}_c, \qquad X_d' = m_l'\frac{l_g'}{l'}\ddot{x}_d,$$

$$Y_c' = m_l'\left(\frac{l - l_g'}{l'}\right)\ddot{y}_c, \qquad Y_d' = m_l'\frac{l_g'}{l'}\ddot{y}_d.$$

The constraint reaction is S', so that for the kinetic equilibrium of the block, m_d, we have,

$$P - (S'\cos\psi + X_d' + K'\sin\psi) = m_d\ddot{x}_d,$$

$$\therefore \quad S' = \frac{P - m_d\ddot{x}_d - X_d' - K'\sin\psi}{\cos\psi}.$$

The total components at the pin c are

$$\begin{aligned} X_c &= X_c' - S'\cos\psi - K'\sin\psi \\ &= X_c' + X_d' + m_d\ddot{x}_d - P, \\ Y &= Y_c' + S'\sin\psi - K'\cos\psi \\ &= Y_c' + (P - m_d\ddot{x}_d - X_d' - K'\sin\psi)\tan\psi - K\cos\psi. \end{aligned}$$

If N is the normal component of the reaction between the moving block and oscillating lever the motion of the

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* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 365, vol. 210, September, 1930.

oscillating lever is

$$Nl_{ab} - X_c l \cos \phi + Y_c l \sin \phi = I_l \ddot{\phi},$$

$$\therefore \quad N = \frac{1}{l_{ab}} (X_c l \cos \phi - Y_c l \sin \phi + I_l \ddot{\phi})$$

Multiplying by $l_{ab} \cdot K_{\phi\theta}$ and substituting the value of N obtained previously from the consideration of the crank and block,

$$\frac{- P_\theta \cos (\theta - \phi) - (m_b r^2 + I_r) \cos (\theta - \phi) \ddot{\theta}}{\cos (\theta - \phi) + \mu \sin (\theta - \phi)}$$
$$= (X_c' + X_d' + m_d \ddot{x}_d - P) l \cos \phi \cdot K_{\phi\theta}$$
$$- (Y_c' + (P - m_d \ddot{x}_d - X_d' - K \sin \psi) \tan \psi$$
$$- K \cos \psi) l \sin \phi \cdot K_{\phi\theta} + I_l K_{\phi\theta} \ddot{\phi}.$$

Substituting the values for X_d', X_d', etc., and simplifying, we obtain

$$\frac{- P_\theta \cos (\theta - \phi)}{\cos (\theta - \phi) + \mu \sin (\theta - \phi)} + PK_{xd\theta}$$
$$= \frac{(m_b r^2 + I_r) \cos (\theta - \phi) \ddot{\theta}}{\cos (\theta - \phi) + \mu \sin (\theta - \phi)} + I_l K_{\phi\theta} \ddot{\phi} + I_l' K_{\psi\theta} \ddot{\psi}$$
$$+ m_l' K_{xc\theta} \ddot{x}_c + m_l' K_{yc\theta} \ddot{y}_c + m_d \ddot{x}_d K_{xd\theta}$$
$$+ m_l' \frac{l_g'}{l'} (K_{xd\theta} \ddot{x}_d - K_{xc\theta} \ddot{x}_c - K_{yc\theta} \ddot{y}_c - l'^2 K_{\psi\theta} \ddot{\psi}).$$

which is the generalized moment equation of the system.

Since

$$\ddot{x}_d = \ddot{x}_c + l' \psi \cos \psi^2 + l' \ddot{\psi} \sin \psi$$

and

$$K_{xd\theta} = \frac{l \cos (\phi - \psi)}{\cos \psi} K_{\phi\theta}, \qquad K_{xc\theta} = l \cos \phi \cdot K_{\phi\theta},$$

$$K_{\psi\theta} = \frac{l \sin \phi}{l' \cos \psi} \cdot K_{\phi\theta}.$$

The last term of this expression can be expressed also in the form,

$$m_l' l_g' [K_{\psi\theta} \sin \psi \cdot \ddot{x}_c + (K_{xc\theta} \cos \psi - K_{yc\theta} \sin \psi) K_{\psi\theta} \dot{\theta}^2$$
$$+ (K_{xc\theta} \sin \psi + K_{yc\theta} \cos \psi) \ddot{\psi} + K_{\psi\theta} \cos \psi \cdot \ddot{y}_c]$$

and, substituting the values

$$\ddot{x}_c = K_{xc\theta}\ddot{\theta} + \frac{dK_{xc\theta}}{d\theta}\cdot\dot{\theta}^2, \qquad \ddot{y}_c = K_{yc\theta}\ddot{\theta} + \frac{dK_{yc\theta}}{d\theta}\cdot\dot{\theta}^2,$$

$$\ddot{\psi} = K_{\psi\theta}\ddot{\theta} + \frac{dK_{\psi\theta}}{d\theta}\cdot\dot{\theta}^2,$$

we obtain the expression previously derived directly from the generalized equation of motion.

To obtain the angular velocity $\dot{\theta}$ for any particular configuration θ, we have

$$\tfrac{1}{2}A(\dot{\theta}^2 - \dot{\theta}_0{}^2) = \int_{\theta_0}^{\theta}(-P_\theta + PK_{xd\theta} - \mu NK_{\mathrm{rel}\,\theta})d\theta$$

as the reaction N involves the acceleration $\ddot{\theta}$, we may allow for the work of friction by an assumed efficiency constant ϵ, so that

$$\tfrac{1}{2}A(\dot{\theta}^2 - \theta_0{}^2) = \epsilon\int_{\theta_0}^{\theta}(-P_\theta + PK_{xd\theta})d\theta,$$

from which we may obtain $\dot{\theta}$ with sufficient approximation. Then from the equation of motion we can calculate $\ddot{\theta}$ for the given configuration.

RESULTANT REACTIONS ON THE FRAME.

The reactions on the frame consists of the component reactions at the shaft, the pivot reaction of the oscillating lever, and the normal component of the upper sliding block on its guide. Let

X_r and Y_r = the component reactions at the crank shaft.
X_a and Y_a = the component reactions at the pivot of the oscillating lever.
N_d = normal component reaction on the upper moving block m_d.
P_f = torque reaction of frame on the linkage system about the crank shaft.
A.M. = angular momentum of system about the crank shaft axis.

Evidently,

$$\begin{aligned}\text{A.M.} = (I_r + m_b r^2)\dot{\theta} + (I_l - m_l l_g h \cos\phi)\phi \\ + m'[(l\cos\phi - h)\dot{x}_c - l\sin\phi\cdot\dot{y}_c \\ + l_g'(y_c\sin\psi - x_c\cos\psi)\psi \\ + l_g'(\dot{x}_c\sin\psi + \dot{y}_c\cos\psi)].\end{aligned}$$

The resultant frame reactions consist of the components'

$$\sum X_f = X_r + X_a, \qquad \sum Y_f = Y_r + Y_a + N_d$$

and

$$P_f = -N_d(l\sin\phi - l'\cos\psi) - X_c h.$$

The equations of linear and angular momentum are

$$\begin{aligned}\sum X_f + P = m_l(l_g\ddot{\phi}\cos\phi - l_g\phi^2\sin\phi) \\ + m_l'(\ddot{x}_c + l_g'\ddot{\psi}\sin\psi + l_g'\psi^2\cos\psi) \\ + m_d\ddot{x}_d + m_b(r\ddot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta),\end{aligned}$$

$$\begin{aligned}\sum Y_f = m_l(-l_g\ddot{\phi}\sin\phi - l_g\phi^2\cos\phi) \\ + m_l'(\ddot{y}_c + l_g'\ddot{\psi}\cos\psi - l_g'\psi^2\sin\psi) \\ - m_b(r\ddot{\theta}\sin\theta + r\dot{\theta}^2\cos\theta)\end{aligned}$$

$$\begin{aligned}P_f - P_\theta + P(l\cos\phi + l'\sin\psi) \\ = (I_r + m_b r^2)\ddot{\theta} + (I_l - m_g l_g h\cos\phi)\ddot{\phi} + I_l'\ddot{\psi} \\ + m_l'[(l\cos\phi - h)\ddot{x}_c - l\sin\phi\cdot\ddot{y}_c \\ + l_g'(y_c\sin\psi - x_c\cos\psi)\ddot{\psi} \\ + l_g'(\ddot{x}_c\sin\psi + \ddot{y}_c\cos\psi) \\ - l\sin\phi\,\phi\dot{x}_c - l\cos\phi\,\phi\dot{y}_c \\ + l_g'(y_c\cos\psi + x_c\sin\psi)\psi^2 + m_l l_g h\sin\phi\cdot\phi^2,\end{aligned}$$

where $-P_\theta$ is the torque exerted on the crank shaft and P is the resistance along the upper slide block at d. Evidently $\sum X_f$, $\sum Y_f$, P_f, $-P_\theta$ and P are the external reactions acting on the moving parts, the friction force μN, etc., being internal reactions and therefore not affecting the motion of the c. of g. or the angular momentum of the system as a whole.

Substituting for the velocities and accelerations, the velocity ratios and their derivatives, we have for the resultant reactions on the frame,

$$
\begin{aligned}
-\sum X_f = {} & P - [m_l l_g K_{\phi\theta} \cos\phi + m_l'(K_{xc\theta} + l_g' K_{\psi\theta} \sin\psi) \\
& + m_d K_{xd\theta} + m_b r \cos\theta]\ddot{\theta} - \biggl[m_l l_g \biggl(\frac{dK_{\phi\theta}}{d\theta} \cos\phi \\
& - K_{\phi\theta}{}^2 \sin\phi \biggr) + m_l' \biggl(\frac{dK_{xc\theta}}{d\theta} + l_g' \biggl(\frac{dK_{\psi\theta}}{d\theta} \sin\psi \\
& + K_{\psi\theta} \cos\psi \biggr)\biggr) + m_d \frac{dK_{xd\theta}}{d\theta} - m_b r \sin\theta \biggr] \dot{\theta}^2, \\
-\sum Y_f = {} & [m_l l_g K_{\phi\theta} \sin\phi - m_l'(K_{yc\theta} + l_g' K_{\psi\theta} \cos\psi) \\
& - m_b r \sin\theta]\ddot{\theta} + \biggl[m_l l_g \biggl(\frac{dK_{\phi\theta}}{d\theta} \sin\phi \\
& + K_{\phi\theta}{}^2 \cos\phi \biggr) - m_l' \biggl(\frac{dK_{yc\theta}}{d\theta} + l_g' \frac{dK_{\psi\theta}}{d\theta} \cos\psi \\
& - l_g' K_{\psi\theta}{}^2 \sin\psi \biggr) + m_b r \cos\theta \biggr] \dot{\theta}^2,
\end{aligned}
$$

$$
\begin{aligned}
-P = {} & -P_\theta + P(l\cos\phi + l'\sin\psi) - [(I_r + m_b r^2) \\
& + (I_l - m_g l_g h \cos\phi) K_{\phi\theta} + I_l' K_{\psi\theta} \\
& + m_l' \{ (l\cos\phi - h + l_g' \sin\psi) K_{xc\theta} \\
& - (l\sin\phi - l_g' \cos\psi) K_{yc\theta} + l_g'(y_c \sin\psi \\
& - x_c \cos\psi) K_{\psi\theta} \}]\ddot{\theta} - \biggl[(I_l - m_g l_g h \cos\phi) \frac{dK_{\phi\theta}}{d\theta} \\
& + I_l \frac{dK_{\psi\theta}}{d\theta} + m' \biggl\{ (l\cos\phi - h + l_g' \sin\psi) \frac{dK_{xc\theta}}{d\theta} \\
& - (l\sin\phi - l_g' \cos\psi) \frac{dK_{yc\theta}}{d\theta} + l_g'(y_c \sin\psi \\
& - x_c \cos\psi) \frac{dK_{\psi\theta}}{d\theta} - l(\sin\phi K_{xc\theta} + \cos\theta K_{yc\theta}) K_{\phi\theta} \\
& + l_g'(y_c \cos\psi + x_c \sin\psi) K_{\psi\theta}{}^2 \biggr\} - m_g l_g h \sin\phi K_{\phi\theta}{}^2 \biggr] \dot{\theta}^2,
\end{aligned}
$$

where $x_c = l \sin\phi$ and $y_c = l \cos\phi$. The acceleration $\ddot{\theta}$ and velocity $\dot{\theta}$ are obtained from the previously derived generalized equation of motion and the energy equation respectively.

GRAPHICAL PROCEDURE AND EXPANSION OF A DYNAMIC SYSTEM INTO A HARMONIC SERIES.

To establish the equations of motion we are concerned with the kinematic data of velocity ratios and their derivatives. We first plot the velocity ratios with respect to the given coördinate. By assuming the fundamental coördinate at unity, the velocity ratios are the numerical value of the actual velocities of the members. We may also derive the velocity ratios from the displacement diagrams of the members when plotted against the primary coördinate. The derivative of the displacement diagrams gives the velocity ratio. Thus, if the displacement diagram is a simple harmonic function of the coördinate, the velocity ratio diagram is displaced 90° from its corresponding displacement diagram. An irregular or complex curve for the displacement diagram may to advantage in many cases be reduced by Fourier's series to a series of harmonics from which the corresponding component velocity diagrams may be obtained.

From the subnormal of the velocity ratio diagram we obtain the values $K\frac{dK}{d\theta}$ which are required for the centrifugal components in the generalized equations of motion. For the centrifugal generalized accelerations, the $\frac{dK}{d\theta}$'s, may be obtained either by dividing the $K\frac{dK}{d\theta}$ curves by K or by directly estimating the derivatives of the velocity ratio diagrams plotted against θ.

A semi-analytical-graphical procedure would be by the resolution of the velocity ratios plotted against the primary coördinate. Then, an analysis of the velocity ratio curve into a harmonic series by Fourier's theorem. The $\frac{dK}{d\theta}$'s and $K\frac{dK}{d\theta}$'s are obtained analytically, which with the inertia constants of the system enables us to obtain the equations of motion, etc. The velocity ratios also may be plotted from their analytical values and then expanded into a Fourier's series.

The interesting point is that, the dynamical equations of a system may thus be expanded into a harmonic series, the amplitudes for the various harmonics depending also on the various inertia properties of the members.

As an example of the graphical procedure, the velocity curves and their derivatives were plotted for the Turning Block Linkage, Fig. 67, previously discussed. Only a portion of the curves are shown.

Fig. 68*a*, gives the $K_{xb\theta}$, $K_{xb\theta}\dfrac{dK_{xb\theta}}{d\theta}$, and $\dfrac{dK_{xb\theta}}{d\theta}$, for the x component of the trunion of the moving slide block "*b*."

Fig. 68*b*, gives the $K_{yb\theta}$, $K_{yb\theta}\dfrac{dK_{yb\theta}}{d\theta}$, and $\dfrac{dK_{yb\theta}}{d\theta}$, for the y component of "*b*."

Fig. 68*c*, gives the $K_{\phi\theta}$, $K_{\phi\theta}\dfrac{dK_{\phi\theta}}{d\theta}$, and $\dfrac{dK_{\phi\theta}}{d\theta}$, for the oscillating lever "*l*."

ON DEGREES OF FREEDOM AND REACTIONS OF CONSTRAINT.

With systems of more than one degree of freedom, we may employ Lagrange's equations to advantage since we have a dynamical equation expressed as partial variations of the kinetic energy of each independent coördinate. Since the Lagrangian method is virtually an extension of the D'Alembert dynamical equation to generalized coördinates, it, of course, enjoys all the advantages of the latter equations in the form of an extensive elimination of reactions.

In the ordinary process of direct analysis we must have $3n$ equations for each particle and m equations for the constraints, so that we have in all $3n + m$ equations for the solution of the coördinates and reactions of the constraint. If now, the system becomes considerably constrained, we reduce the degrees of freedom to $3n - m$ independent coördinates or degrees of freedom. Thus the Lagrangian method only requires the solution of $3n - m$ equations as against the $3n + m$ equations for a solution by the ordinary equations of motion. The assumption of rigidity of the given parts, of course, greatly reduces the required number of ordinary equations

Curve of $k_{xb\theta}$, $k_{xb\theta}\frac{dk}{d\theta}{}_{xb\theta}$ and $\frac{dk}{d\theta}{}_{xb\theta}$

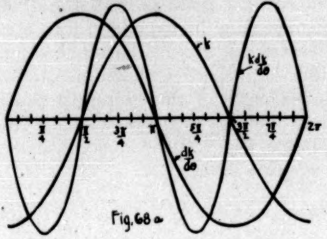


Fig. 68a

Curve of $k_{yb\theta}$, $k_{yb\theta}\frac{dk}{d\theta}{}_{yb\theta}$ and $\frac{dk}{d\theta}{}_{yb\theta}$

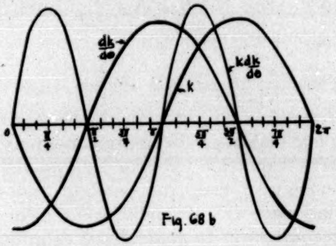


Fig. 68b

Curve of $k_{\phi\theta}$, $k_{\phi\theta}\frac{dk}{d\theta}{}_{\phi\theta}$ and $\frac{dk}{d\theta}{}_{\phi\theta}$.

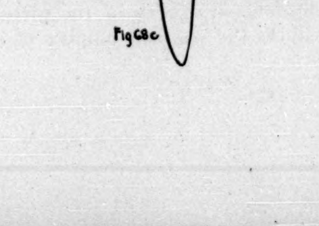


Fig 68c

of motion, so that in many problems in machine analysis the direct or Newtonian method is only slightly more laborious than by the method of Lagrange.

It is important to note with systems of two degrees of freedom we may combine the energy equation, that is the integral of the equation of motion, with that of either the linear or angular momentum. This serves often as an excellent check in the many problems that occur in engineering analysis with two degrees of freedom.

In a constrained system, therefore, by eliminating the conditions of constraint the system can be expressed as a function of the independent coördinates. We are thus actually eliminating the corresponding constraint reactions. If, however, we choose some dependent coördinates in defining the configuration of the system we must then include the corresponding reactions of constraints. In machine analysis we are often concerned with the nature of the internal reactions and the reactions of constraint.

The most simple case is that of a mass particle constrained by a surface. The equation of the surface can be expressed by $z = f(x, y)$. The particle has two degrees of freedom, x and y. Let X, Y, and Z be the applied forces. The work of the applied forces is

$$X\delta x + Y\delta y + Z\delta z = \left(X + Z\frac{\partial z}{\delta x}\right)\delta x + \left(Y + Z\frac{\partial z}{\partial y}\right)\delta y,$$

the work of the reactions of constraint being nil. The generalized forces corresponding to the coördinates x and y are, therefore,

$$X + Z\frac{\partial z}{\partial x} \quad \text{and} \quad Y + Z\frac{\partial z}{\partial y}.$$

The kinetic energy is

$$T = \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \tfrac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \left(\frac{\partial z}{\partial x}\dot{x} + \frac{\partial z}{\partial y}\dot{y}\right)^2\right),$$

so that

$$\frac{\partial T}{\partial \dot{x}} = m\left(\dot{x} + \left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\,\dot{y} \right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m\left[\ddot{x} + 2\frac{\partial z}{\partial x}\left(\frac{\partial^2 z}{\partial x^2}\dot{x}^2 + \frac{\partial^2 z}{\partial x \partial y}\dot{x}\dot{y}\right) + \left(\frac{\partial z}{\partial x}\right)^2 \ddot{x} \right.$$
$$+ \frac{\partial^2 z}{\partial x^2}\cdot\frac{\partial z}{\partial y}\dot{x}\dot{y} + \frac{\partial^2 z}{\partial x \partial y}\cdot\frac{\partial z}{\partial y}\cdot\dot{y}^2 + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 z}{\partial x \partial y}\dot{x}\dot{y}$$
$$\left. + \frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\ddot{y} \right],$$

$$\frac{\partial T}{\partial x} = \left(\frac{\partial z}{\partial x}\dot{x} + \frac{\partial z}{\partial y}\dot{y}\right)\left(\frac{\partial^2 z}{\partial x^2}\cdot\dot{x} + \frac{\partial^2 z}{\partial x \partial y}\cdot\dot{y}\right).$$

The equation of motion corresponding to the x coördinate is, therefore,

$$m\left(\ddot{x} + \left(\frac{\partial z}{\partial x}\right)^2 \ddot{x} + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\ddot{y} + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 z}{\partial x^2}\dot{x}^2 + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 z}{\partial y^2}\dot{y}^2 \right.$$
$$\left. + 2\frac{\partial z}{\partial x}\cdot\frac{\partial^2 z}{\partial x \partial y}\dot{x}\dot{y} \right) = X + Z\frac{\partial z}{\partial x}.$$

To analyze the nature of the constraint reaction we have the condition equation,

$$\delta z - \frac{\partial z}{\delta x}\partial x - \frac{\partial z}{\partial y}\delta y = 0, \qquad (1)$$

with D'Alembert's equation,

$$(X - m\ddot{x})\delta x + (Y - m\ddot{y})\delta y + (Z - m\ddot{z})\delta z = 0, \qquad (2)$$

where δx, δy, and δz are not independent but are subjected to the condition relation given in (1). Multiplying (1) by the multiplier we have the work of the reaction of constraint given by

$$\lambda\left(\delta z - \frac{\partial z}{\partial x}\delta x - \frac{\partial z}{\partial y}\delta y \right) = 0,$$

so that the constraint reactions corresponding to the coördinates x, y, and z are

$$-\lambda\frac{\partial z}{\partial x}, \qquad -\lambda\frac{\partial z}{\partial y}, \qquad \text{and} \qquad \lambda.$$

By introducing the reactions of constraint in equation 2, the coördinates x, y, and z become arbitrary, that is,

$$\left(X - m\ddot{x} - \lambda\frac{\partial z}{\partial x}\right)\delta x + \left(Y - m\ddot{y} - \lambda\frac{\partial z}{\partial y}\right)\delta y + (Z - m\ddot{z} + \lambda)\delta z = 0,$$

so that

$$X - m\ddot{x} - \lambda\frac{\partial z}{\partial x} = 0, \quad Y - m\ddot{y} - \lambda\frac{\partial z}{\partial y} = 0, \quad Z - m\ddot{z} + \lambda = 0.$$

Therefore, $\lambda = m\ddot{z} - Z$, and

$$X + Z\frac{\partial z}{\partial x} = m\left(\ddot{x} + \ddot{z}\frac{\partial z}{\partial x}\right),$$

$$Y + Z\frac{\partial z}{\partial y} = m\left(\ddot{y} + \ddot{z}\frac{\partial z}{\partial y}\right).$$

Since

$$\dot{z} = \frac{\partial z}{\partial x}\dot{x} + \frac{\partial z}{\partial y}\dot{y},$$

$$\ddot{z} = \frac{\partial z}{\partial x}\ddot{x} + \frac{\partial z}{\partial y}\ddot{y} + \frac{\partial^2 z}{\partial x^2}\dot{x}^2 + \frac{\partial^2 z}{\partial y^2}\dot{y}^2 + 2\frac{\partial^2 z}{\partial x\partial y}\dot{x}\dot{y},$$

we have

$$X + Z\frac{\partial z}{\partial x} = m\left(\ddot{x} + \left(\frac{\partial z}{\partial x}\right)^2\ddot{x} + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\ddot{y} + \frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x^2}\dot{x}^2 + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 z}{\partial y^2}\dot{y}^2 + 2\frac{\partial z}{\partial x}\cdot\frac{\partial^2 z}{\partial x\partial y}\dot{x}\dot{y}\right),$$

$$Y + Z\frac{\partial z}{\partial y} = m\left(\ddot{y} + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\ddot{x} + \left(\frac{\partial z}{\partial y}\right)^2\ddot{y} + \frac{\partial z}{\partial y}\cdot\frac{\partial^2 z}{\partial x^2}\dot{x}^2 + \frac{\partial z}{\partial y}\cdot\frac{\partial^2 z}{\partial y^2}\dot{y}^2 + 2\frac{\partial z}{\partial y}\cdot\frac{\partial^2 z}{\partial x\partial y}\dot{x}\dot{y}\right),$$

from which we may calculate $\ddot{x}$ and $\ddot{y}$. We note the agreement with the previous derivation.

To calculate the reaction of constraint of the surface N, we note

$$N_x = -\lambda\frac{\partial z}{\partial x}, \qquad N_y = -\lambda\frac{\partial z}{\partial y}, \qquad N_z = \lambda,$$

or

$$N = \lambda\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2},$$

where

$$\lambda = m\ddot{z} - Z.$$

ANALYSIS OF FRAME SYSTEM WITH TWO DEGREES OF FREEDOM.

A simple statical problem worthy of consideration is an assumed rigid frame shown in Fig. 69. To determine the thrusts in bars BF and CE, we may replace the bars by the corresponding equal and opposite reactions exerted at B and F and at C and E on the remaining frame. Let w_1, w_2, and w_3 be the weights of the upper middle and lower bars respectively, while a, b, and c are their corresponding lengths. The system considered has then two degrees of freedom, θ and ϕ, that is, the angles made by the upper bars and lower bars with respect to the vertical.

Assume the vertical axis y downward and the lateral or horizontal axis x. If T_1 and T_2 are the thrusts of BF and CE we have, in general,

$$\sum T\frac{\partial x}{\partial \theta}\delta\theta + \sum T\frac{\partial x}{\partial \phi}\delta\phi - \frac{\partial V}{\partial \theta}\delta\theta - \frac{\partial V}{\partial \phi}\delta\phi = 0,$$

that is,

$$\sum mg\frac{dy}{\partial \theta} + \sum T\frac{\partial x}{\partial \theta} = 0, \tag{1}$$

$$\sum mg\frac{dy}{\partial \phi} + \sum T\frac{\partial x}{\partial \phi} = 0, \tag{2}$$

which are the conditions of equilibrium. Evidently,

$$x_1 = 2a\sin\theta, \qquad \frac{\partial x_1}{\partial \theta} = 2a\cos\theta, \qquad \frac{\partial x_1}{\partial \phi} = 0,$$

$$x_2 = 2c\sin\phi, \qquad \frac{\partial x_2}{\partial \phi} = 2c\cos\phi, \qquad \frac{\partial x_2}{\partial \theta} = 0 \text{ (approx.)};$$

also

$$y_1 = \frac{a}{2}\cos\theta, \qquad \frac{\partial y_1}{\partial\theta} = -\frac{a}{2}\sin\theta, \quad \frac{\partial y_1}{\partial\phi} = 0,$$

$$y_2 = a\cos\theta + \frac{b}{2}, \qquad \frac{\partial y_2}{\partial\theta} = -a\sin\theta, \quad \frac{\partial y_2}{\partial\phi} = 0,$$

$$y_3 = a\cos\theta + b + \frac{c}{2}\cos\phi, \quad \frac{\partial y_3}{\partial\theta} = -a\sin\theta, \quad \frac{\partial y_3}{\partial\phi} = -\frac{c}{2}\sin\phi.$$

Substituting in (1) and (2),*

$$-(w_1 + 2w_2 + 2w_3)a\sin\theta + 2T_1 a\cos\theta = 0, \tag{3}$$

$$-w_3c\sin\phi + 2T_2c\cos\phi = 0; \tag{4}$$

$$\therefore\quad T_1 = \left(\frac{w_1 + 2w_2 + 2w_3}{2}\right)\tan\phi, \qquad T_2 = \frac{w_3}{2}\tan\phi.$$

ANALYSIS OF ELEMENTARY MECHANICAL SYSTEMS WITH TWO OR MORE DEGREES OF FREEDOM.

We will now consider some simple mechanical systems with special emphasis as to the analysis of the reactions.

We will first consider a problem given by Routh of a rod turning about a hinge at one end and the other end resting on a smooth wedge, which can slide along a smooth horizontal plane.

Let (Fig. 70)

l = length of rod, θ = angle of rod with plane,
α = angle of wedge, x = displacement of wedge,
m = the mass of the rod with radius of gyration k about the hinge,
M = the mass of the wedge.

The coördinates of the system are x and θ, but these are not independent, being connected by the relation

$$F = x\sin\alpha - l\sin(\alpha - \theta) = 0.$$

---

* The weight of rods BF and CE are neglected. When included, we have axial thrust components T_1 and T_2 and lateral components $Y_1 = w_{bf/2}$, $Y_2 = w_{ce/2}$ respectively, acting at their ends. We must then include the additional virtual works $2Y_1\delta(a\cos\theta) = -w_{bf}\; a\sin\theta\cdot\delta\theta$ and $2Y_2\delta(a\cos\theta + b) = -w_{ce}\cdot a\sin\theta\cdot\delta\theta$, so that T_1 and T_2 will be increased.

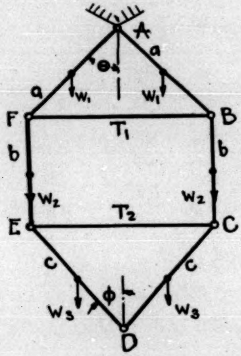

A
a
θ
a
w₁
w₁
F
B
T₁
b
b
W₂
W₂
E
T₂
C
c
c
φ
W₃
W₃
D

fig. 69

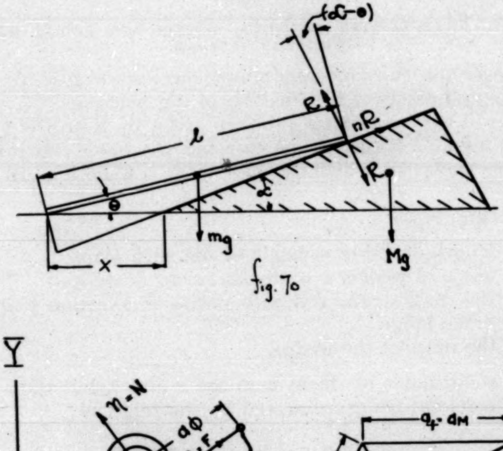

(α−θ)
R
nR
l
R
θ
α
mg
Mg
x

fig. 70

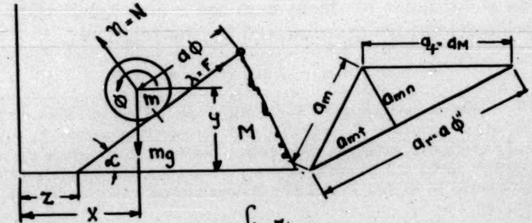

Y
η=N
aφ
λ=F
φ
m
y
M
α
mg
z
x
$a_f = a_M$
a_m
a_{mn}
a_{mt}
$a_r = a\phi''$

fig. 71

By this condition relation the system can be reduced to one degree of freedom. Therefore,

$$T = \tfrac{1}{2}(M\dot{x}^2 + mk^2\dot{\theta}^2) = \tfrac{1}{2}\left(Ml^2 \frac{\cos^2 (\alpha - \theta)}{\sin^2 \alpha} + mk^2 \right) \dot{\theta}^2,$$

so that the coefficient of inertia

$$A = Ml^2 \frac{\cos^2 (\alpha - \theta)}{\sin^2 \alpha} + mk^2$$

is a function of θ. For the potential,

$$V = mg\frac{l}{2}\sin \theta + c, \qquad \frac{\partial V}{\partial \theta} = mg\frac{l}{2}\cos \theta.$$

From the Lagrangian equation we have

$$\frac{\partial T}{\partial \dot{\theta}} = \left(Ml^2 \frac{\cos^2 (\alpha - \theta)}{\sin^2 \alpha} + mk^2 \right) \dot{\theta},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = A\ddot{\theta} + 2Ml^2 \left(\frac{\sin (\alpha - \theta) \cos (\alpha - \theta)}{\sin^2 \alpha} \right) \dot{\theta}^2,$$

$$\frac{\partial T}{\partial \theta} = Ml^2 \left(\frac{\sin (\alpha - \theta) \cos (\alpha - \theta)}{\sin^2 \alpha} \right) \dot{\theta}^2,$$

so that the equation of motion is

$$A\ddot{\theta} + Ml^2 \frac{\sin 2(\alpha - \theta)}{2 \sin^2 \alpha} \cdot \dot{\theta}^2 = - mg\frac{l}{2}\cos \theta.$$

This is also evident from the generalized equation of motion for one degree of freedom,

$$A\ddot{\theta} + \frac{1}{2}\frac{dA}{d\theta} \cdot \dot{\theta}^2 = - \frac{\partial V}{\partial \theta},$$

where

$$\frac{1}{2}\frac{dA}{d\theta} = \frac{Ml^2 \cos^2 (\alpha - \theta)}{2 \sin^2 \alpha}.$$

Here the method of velocity ratios offers a simplification in the kinematic data.

Resolving normally to inclined plane of wedge,

$$l\dot{\theta}\cos(\alpha - \theta) = -\dot{x}\sin\alpha;$$

$$\therefore\quad K_{x\theta} = -\frac{l\cos(\alpha - \theta)}{\sin\alpha}, \qquad \frac{dK_{x\theta}}{d\theta} = -\frac{l\sin(\alpha - \theta)}{\sin\alpha}.$$

Substituting in the equation of motion,

$$(mk^2 + MK_{x\theta}^2)\ddot{\theta} + MK_{x\theta}\frac{dK_{x\theta}}{d\theta}\cdot\dot{\theta}^2 = -\frac{\partial V}{\partial\theta},$$

we obtain the previous results.

To obtain the velocity we use the equation of energy. If θ_0 is the initial angle of rod, then

$$\frac{1}{2}A(\dot{\theta}^2 - \dot{\theta}_0^2)_0 = mg\frac{l}{2}(\sin\theta_0 - \sin\theta).$$

To determine the reaction between the rod and wedge we will now consider the system as having two degrees of freedom x and θ. But these are not independent, being connected by the relation

$$F = x\sin\alpha - l\sin(\alpha - \theta) = 0.$$

Therefore, in the equations of Lagrange we must include the reaction of constraint imposed by F,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{x}}\right) - \frac{\partial T}{\partial x} = X - \frac{\partial V}{\partial x} + \lambda\frac{\partial F}{\partial x},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) - \frac{\partial T}{\partial\theta} = \Theta - \frac{\partial V}{\partial\theta} + \lambda\frac{\partial F}{\partial\theta},$$

where

$$T = \tfrac{1}{2}(M\dot{x}^2 + mk^2\dot{\theta}^2),$$

$$V = mg\frac{l}{2}\sin\theta + c, \qquad \frac{\partial V}{\partial\theta} = mg\frac{l}{2}\cos\theta,$$

$$X = 0,\ \Theta = 0, \qquad \frac{\partial V}{\partial x} = 0,$$

$$\frac{\partial T}{\partial\dot{x}} = M\dot{x}, \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial\dot{x}}\right) = M\ddot{x}, \qquad \frac{\partial T}{\partial x} = 0,$$

$$\frac{\partial T}{\partial\dot{\theta}} = mk^2\dot{\theta}, \qquad \frac{\partial}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) = mk^2\ddot{\theta}, \qquad \frac{\partial T}{\partial\theta} = 0;$$

also, since $F = x \sin \alpha - l \sin (\alpha - \theta) = 0$,

$$\frac{\partial F}{\partial x} = \sin \alpha, \qquad \frac{\partial F}{\partial \theta} = l \cos (\alpha - \theta),$$

$$\ddot{x} = -\frac{l}{\sin \alpha}(\cos (\alpha - \theta)\ddot{\theta} + \sin (\alpha - \theta) \cdot \dot{\theta}^2),$$

so that the equations of motions are

$$M\ddot{x} = \lambda \sin \alpha,$$

$$mk^2\ddot{\theta} = -mg\frac{l}{2}\cos \theta + \lambda l \cos (\alpha - \theta).$$

Eliminating λ and substituting the above value for $\ddot{x}$, we obtain the previous equation of motion.

We note the reaction corresponding to the connection is the mutual reaction between the wedge and rod and normal to the inclined plane of the wedge. That is,

$$\left.\begin{aligned} R \sin \alpha &= \lambda \sin \alpha, \\ Rl \cos (\alpha - \theta) &= \lambda l \cos (\alpha - \theta); \end{aligned}\right| \quad \therefore \quad \lambda = R.$$

Friction between Rod and Wedge (Fig. 70).—The frictional component opposes the relative motion between any two bodies, A and B, at their point of contact. Since the motion of B relative to A is equal and opposite to A relative to B, the friction reaction on B is equal and opposite to that on A, which is likewise obvious from the principle of action and reaction.

In considering the relative motion or the corresponding frictional forces, themselves, we must consider the actual motion of the system. Since the frictional forces can only oppose the motion to a varying degree we may determine the directions of the relative motion by solving for the motion without friction. In simple systems the motion of the parts is usually obvious.

The mutual reaction between rod and wedge may now be divided into components, μR and R, tangential and normal to the inclined plane of the wedge. The magnitude of R, however, depends upon the inertia forces of the system, i.e., we have

$$R(\cos(\alpha-\theta)+\mu\sin(\alpha-\theta))l - mg\frac{l}{2}\cos\theta = mk^2\ddot{\theta};$$

$$\therefore\quad R = \frac{mg\frac{l}{2}\cos\theta + mk^2\ddot{\theta}}{l(\cos(\alpha-\theta)+\mu\sin(\alpha-\theta))}.$$

The work of the frictional component obviously depends only on its relative displacement and is

$$-\mu R(\delta x\cos\alpha - l\delta\theta\sin(\alpha-\theta)),$$

or introducing the condition relation,

$$\mu R\left(\frac{\cos(\alpha-\theta)}{\tan\alpha}+\sin(\alpha-\theta)\right)l\delta\theta,$$

where the former relation is of use when we consider the system with two degrees of freedom and the latter for one degree of freedom.

The virtual work of the normal component is

$$Rl\cos(\alpha-\theta)\delta\theta + R\sin\alpha\delta x,$$

or introducing the condition relation,

$$Rl\cos(\alpha-\theta)\delta\theta - Rl\frac{\cos(\alpha-\theta)}{\sin\alpha}\cdot\sin\alpha\cdot\delta\theta = 0.$$

That is, for the system as a whole the normal component does no work.

Expressed in the equation of motion for one degree of freedom, we have

$$A\ddot{\theta} + ml^2\frac{\sin 2(\alpha-\theta)}{2\sin^2\alpha}\cdot\dot{\theta}^2$$
$$= -mg\frac{l}{2}\cos\theta + \mu R\left(\frac{\cos(\alpha-\theta)}{\tan\alpha}+\sin(\alpha-\theta)\right)l$$

Substituting for R and simplifying, we obtain

$$\left[A - \frac{\mu mk^2\cos\theta}{\sin\alpha(\cos(\alpha-\theta)+\mu\sin(\alpha-\theta))}\right]\ddot{\theta}$$
$$+ Ml^2\frac{\sin 2(\alpha-\theta)}{2\sin^2\alpha}\cdot\dot{\theta}^2$$
$$= -mg\frac{l}{2}\cos\theta\left[1 - \frac{\mu\cos\theta}{\sin\alpha(\cos(\alpha-\theta)+\mu\sin(\alpha-\theta))}\right].$$

Expressed in the equation of motion for two degrees of freedom,

$$M\ddot{x} = R \sin \alpha - \mu R \cos \alpha,$$

$$mk^2\ddot{\theta} = - mg\frac{l}{2} \cos \theta + Rl \cos (\alpha - \theta) + \mu Rl \sin (\alpha - \theta),$$

which are the equations of motion for the parts. Eliminating R we can verify the expression above on substituting for $\ddot{x}$ in terms of θ.

ON ROLLING WHEELS ON MOVING FRAMES.

We will further consider some elementary machine combinations, involving rolling of disks on moving frames, to illustrate with simple examples the use of velocity ratios and the determination of the reactions.

For the case of a sliding wedge with a sphere or disk rolling down the rough face of its inclined plane, the wedge itself sliding on a smooth horizontal plane, Fig. 71, we have an elementary system of two degrees of freedom with coordinates z for the motion of the wedge along the horizontal plane and the relative displacement $s = a\phi$ along the rough inclined plane. Let m and k be the mass and radius of gyration of the sphere and M the mass of the wedge and α the angle of the inclined face with respect to the horizontal.

The kinematical relations are, for "m,"

$$\delta x = \delta z - a\delta\phi \cos \alpha, \qquad \delta y = - a\delta\phi \sin \alpha,$$

and for "M,"

$$\delta x = \delta z.$$

The partial velocity ratios for the rolling disk "m" are, therefore,

$$K_{xz} = 1, \qquad K_{x\phi} = - a \cos \alpha, \qquad K_{\phi\phi} = 1,$$

$$K_{yz} = 0, \qquad K_{y\phi} = - a \sin \alpha,$$

$$\frac{\partial K_{xz}}{\partial z} = \frac{\partial K_{yz}}{\partial z} = \frac{\partial K_{y\phi}}{\partial \phi} = \frac{\partial K_{y\phi}}{\partial \phi} = 0,$$

while the partial velocity ratio for the wedge depends only on the coördinate z and is obviously unity in the x direction and nil in the y direction. The partial velocity ratios could readily be obtained as for one degree of freedom, first holding ϕ constant and then z constant.

The potential energy is $-\delta V = -mg\delta y$;

$$\therefore \quad -\frac{\partial V}{\partial \phi} = mga \sin \alpha, \qquad -\frac{\partial V}{\partial z} = 0.$$

Since we are dealing only with cyclic coördinates, i.e., $\frac{\partial K_{xz}}{\partial z}$ etc., $= 0$, the dynamical equations in terms of velocity ratios simplify to

$$m[K_{xz}^2\ddot{z} + K_{xz}K_{x\phi}\ddot{\phi}] + M\ddot{z} = 0,$$
$$m[(K_{x\phi}^2 + K_{y\phi}^2)\ddot{\phi} + K_{x\phi}K_{xz}\ddot{z}] + mk^2\ddot{\phi} = mga \sin \phi.$$

Substituting for the partial velocity ratios, we obtâin

$$m(\ddot{z} - a \cos \alpha \cdot \ddot{\phi}) + M\ddot{z} = 0,$$
$$m(a^2 + k^2)\ddot{\phi} - ma\ddot{z} \cos \alpha = mga \sin \alpha.$$

To obtain the mutual reaction between the sphere and wedge we must consider the motion of the parts separately. We have thus a system with three degrees of freedom, x, y, and ϕ for the sphere and z for the wedge, together with the constraint condition between the parts.

The kinetic and potential energies are

$$T = \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2 + k^2\dot{\phi}^2) + M\dot{z}^2, \qquad V = mgy,$$

with the condition of constraint,

$$F_1 = y \cos \alpha - a - (x - z) \sin \alpha = 0,$$
$$F_2 = (x - z) \cos \alpha + y \sin \alpha + a\phi - L = 0,$$

so that the reactions of constraint are

$$\eta \frac{\partial F_1}{\partial x} = -\eta \sin \alpha, \qquad \eta \frac{\partial F_1}{\partial y} = \eta \cos \alpha,$$
$$\lambda \frac{\partial F_2}{\partial x} = \lambda \cos \alpha, \qquad \lambda \frac{\partial F_2}{\partial y} = \lambda \sin \alpha,$$
$$\lambda \frac{\partial F_2}{\partial \phi} = \lambda a.$$

Substituting in the equations of motion,

$$m\ddot{x} = \lambda \cos \alpha - \eta \sin \alpha, \tag{1}$$

$$m\ddot{y} = \lambda \sin \alpha + \eta \cos \alpha - mg, \tag{2}$$

$$mk^2\ddot{\phi} = \lambda a, \tag{3}$$

$$M\ddot{z} = \eta \sin \alpha - \lambda \cos \alpha. \tag{4}$$

Combining the condition relations of the constraint and differentiating, we obtain

$$\ddot{x} = \ddot{z} - a\ddot{\phi} \cos \alpha, \qquad \ddot{y} = - a\ddot{\phi} \sin \alpha.$$

Combining (1) and (4) and then (1), (2), and (3), we eliminate λ and η. Substituting for the accelerations we obtain the previous equations.

A very simple analysis is by the use of an acceleration diagram, Fig. 71. The sphere rolls relatively to the moving wedge with an acceleration $\bar{a}_R = a\ddot{\phi}$, while the wedge moves with an acceleration $\bar{a}_f = \ddot{z}$. The acceleration of the sphere is then $\bar{a}_f + \bar{a}_R$ adding vectorially.

Let N and F be the normal and tangential components of the reaction between sphere and wedge. The mutual horizontal component between sphere and wedge is

$$H = m(\bar{a}_R \cos \alpha - \bar{a}_f) = M\bar{a}_f;$$

$$\therefore \quad \bar{a}_f = \frac{m\bar{a}_R}{m + M} \cos \alpha.$$

Likewise, for the sphere down the plane,

$$mg \sin \alpha - F = m(\bar{a}_R - \bar{a}_f \cos \alpha),$$

$$Fa = mk^2\ddot{\phi},$$

so that, on combining,

$$\bar{a}_f = \frac{mg \sin \alpha \cos \alpha}{(m + M)\dfrac{k^2}{a^2} + m \sin^2 \alpha + M},$$

which gives the acceleration of the wedge. For the normal component we have

$$N = mg \cos \alpha - m\bar{a}_f \sin \alpha.$$

We will now consider a more complicated problem, Fig. 72. The system consists of two rolling cylinders m_1 and m_2 on the inclined planes of a moving wedge making angles α and β with the horizontal and connected by an inextensible string passing over a pulley of moment of inertia I_m at the apex of the wedge. Let k_1 and k_2 be the radii of gyrations of m_1 and m_2 respectively. We will further suppose the string connecting m_1 and m_2 to be wound around a drum of radius b on m_1. Let a, c, and r be the radius of the cylinders, m_1, m_2, and I_m. Obviously, if the strings are parallel to the planes, $c = a - b$. Finally, let M be the mass of the wedge which is assumed to be free to slide on a frictionless horizontal plane.

The system has two degrees of freedom, z, the horizontal displacement of the wedge, and s, the relative displacement of the string over the inclined faces of the wedge. The angular coördinates of m_1, I_m, and m_2 are, respectively, θ, q, and ϕ, while the coördinates of m_1, m_2, and M are x_1y_1, x_2y_2, and z.

The coördinates are related by the condition equations as follows:

$$a\delta\theta - b\delta\theta = \delta s, \qquad \therefore \quad \delta\theta = \frac{\delta s}{a - b},$$

$$r\delta q = \delta s, \qquad \delta q = \frac{\delta s}{r},$$

$$c\delta\phi = \delta s, \qquad \delta\phi = \frac{\delta s}{c}\,.$$

Also,

$$\left.\begin{aligned} \delta x_1 &= \delta z - a\delta\theta\cos\alpha = \delta z - \frac{a}{a - b}\,\delta s\cos\alpha, \\ \delta y_1 &= -\,a\delta\theta\sin\alpha = -\frac{a}{a - b}\sin\alpha\cdot\delta s, \\ \delta\theta &= \frac{1}{a - b}\,\delta s, \end{aligned}\right| \quad \text{for } m_1,$$

$$\left.\begin{aligned} \delta x_2 &= \delta z - \delta s\cos\beta, \\ \delta y_2 &= \delta s\sin\beta, \qquad \delta\phi = \frac{\delta s}{c}, \\ \delta q &= \frac{\delta s}{r}, \quad \text{for } I_m. \end{aligned}\right| \quad \text{for } m_2.$$

The corresponding partial velocity ratios are

$$\left.\begin{aligned} K_{x_1z} &= 1, & K_{x_1s} &= -\frac{a}{a-b}\cos\alpha, \\ K_{y_1z} &= 0, & K_{y_1s} &= -\frac{a}{a-b}\sin\alpha, \\ K_{\theta z} &= 0, & K_{\theta s} &= \frac{1}{a-b}, \end{aligned}\right| \quad \text{for } m_1,$$

$$\left.\begin{aligned} K_{x_2z} &= 1, & K_{x_2s} &= -\cos\beta, \\ K_{y_2z} &= 0, & K_{y_2s} &= \sin\beta, \\ K_{\phi z} &= 0, & K_{\phi s} &= \frac{1}{c}, \end{aligned}\right| \quad \text{for } m_2,$$

$$K_{qz} = 0, \qquad K_{qs} = \frac{1}{r} \qquad \text{for } I_m.$$

$$K_{zz} = 1, \qquad K_{zs} = 0 \quad \text{for } M.$$

Since the velocity ratios are constant, or nil, their derivatives are zero. That is, we have a cyclic system of coördinates.

The work forces are, since,

$$-m_1g\delta y_1 = -m_1g\frac{\partial y_1}{\partial s}\cdot\delta s,$$

$$-m_2g\delta y_2 = -m_2g\frac{\partial y_2}{\partial s}\cdot\delta s,$$

$$-m_1gK_{y_1s} \quad \text{and} \quad -m_2gK_{y_2s}.$$

The equations of motion, in terms of the velocity ratios, after simplifying, are

$$m_1[K_{x_1z}{}^2\ddot{z} + K_{x_1z}K_{x_1s}\ddot{s}] + m_2[K_{x_2z}{}^2\ddot{z} + K_{x_2z}K_{x_2s}\ddot{s}] + M\ddot{z} = 0$$

and

$$\begin{aligned} m_1[(K_{x_1s}{}^2 + K_{y_1s}{}^2)\ddot{s} + K_{x_1s}K_{x_1z}\ddot{z}] + m_1k_1{}^2K_{\theta s}{}^2\ddot{s} \\ + m_2[(K_{x_2s}{}^2 + K_{y_2s}{}^2)\ddot{s} + K_{x_2s}K_{x_2z}\ddot{z}] + m_2k_2{}^2K_{\phi s}{}^2\ddot{s} \\ + I_mK_{qs}{}^2\ddot{s} = -m_1gK_{y_1s} - m_2gK_{y_2s}. \end{aligned}$$

Substituting for the velocity ratios, we obtain

$$(M + m_1 + m_2)\ddot{z} - \left(m_1 \frac{a}{a - b} \cos \alpha + m_2 \cos \beta \right) \ddot{s} = 0 \qquad (1)$$

and

$$\left(m_1 \frac{a^2 + k_1^2}{(a - b)^2} + m_2 \frac{c^2 + k_2^2}{c^2} + \frac{I_m}{r^2} \right) \ddot{s}$$
$$- \left(m_1 \frac{a}{a - b} \cos \alpha + m_2 \cos \beta \right) \ddot{s}$$
$$= m_1 g \frac{a}{a - b} \sin \alpha - m_2 g \sin \beta. \qquad (2)$$

The equations of motion may be readily verified by Lagrange's equations. For the kinetic energy, we have

$$T = \frac{1}{2} \left[M\dot{z}^2 + m_1 \left(\left(\frac{a}{a - b} \dot{s} - \dot{z} \cos \alpha \right)^2 + \dot{z}^2 \sin^2 \alpha \right) \right.$$
$$+ \frac{m_1 k_1^2 \dot{s}^2}{(a - b)^2} + m_2((\dot{s} - \dot{z} \cos \beta)^2 + \dot{z}^2 \sin^2 \beta)$$
$$\left. + m_2 k_2^2 \frac{\dot{s}^2}{c^2} + I_m \frac{\dot{s}^2}{r^2} \right],$$

and the potential function is

$$V = m_1 g y_1 + m_2 g y_2;$$
$$\therefore \quad \frac{\partial V}{\partial s} = - \left(m_1 g \frac{\partial y_1}{\partial s} + m_2 g \frac{\partial y_2}{\partial s} \right)$$
$$= m_1 g \left(\frac{a}{a - b} \right) \sin \alpha - m_2 g \sin \beta.$$

Since the coördinates are cyclic, i.e.,

$$\frac{\partial T}{\partial s} = \frac{\partial T}{\partial z} = 0,$$

we have

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{s}} \right) = - \frac{\partial V}{\partial s} \qquad \text{and} \qquad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{z}} \right) = 0,$$

which after performing the differentiations agrees with the previous result.

To determine the mutual reactions between the parts we must consider each part separately and introduce the condition relations for the connections between the parts. It is evident, therefore, we must now consider a system with eight degrees of freedom, i.e., $\xi_1 \; \eta_1 \; \theta$ for m_1, $\xi_2 \; \eta_2 \; \phi$ for m_2, q for I_m, and z for M, together with the corresponding constraint connections.

We will assume the absolute coördinates $\xi_1\eta_1$ and $\xi_2\eta_2$ along and normal to the inclined planes of the wedge with positive directions of ξ_1 and ξ_2 the same as the relative displacement of the string.

The equations of connection are (Fig. 72):

(1) For the string between m_1 and I_m and m_2 and I_m,

$$\delta s_1 - r\delta q = 0 \qquad \text{and} \qquad \delta s_2 - r\delta q = 0,$$

where $\delta s_1 = \delta s_2 = \delta s =$ the relative displacement of string over the wedge and q is the angle turned by the pulley I_m at the apex of the wedge.

Equating the absolute displacements of the drum and string at contact of string with drum of cylinder m_1, we have

$$\delta\xi_1 - b\delta\theta = \delta s_1 - \delta z \cos \alpha,$$

so that

$$\delta s_1 = \delta\xi_1 - b\delta\theta + \delta z \cos \alpha.$$

In like manner,

$$\delta s_2 = \delta\xi_2 + \delta z \cos \beta.$$

The work of the string constraint is

$$- \lambda_1(\delta\xi_1 - b\delta\theta + \delta z \cos \alpha - r\delta q), \quad \text{between } m_1 \text{ and } I_m,$$
$$\lambda_2(\delta\xi_2 + \delta z \cos \beta - r\delta q), \quad \text{between } m_2 \text{ and } I_m.$$

(2) For the rolling of the cylinders on the inclined plane of the wedge the virtual work of the friction constraint is

$$\gamma_1(\delta\xi_1 - a\delta\theta + \delta z \cos \alpha) = 0, \quad \text{between } m_1 \text{ and } M,$$
$$- \gamma_2(\delta\xi_2 - c\delta\phi + \delta z \cos \beta) = 0, \quad \text{between } m_2 \text{ and } M.$$

(3) For the condition of contact of cylinders with inclined surfaces of wedge, the work of the normal constraint is

$$\mu_1(\delta\eta_1 + \delta z \sin\alpha) = 0, \quad \text{between } m_1 \text{ and } M,$$

$$\mu_2(\delta\eta_2 - \delta z \sin\beta) = 0, \quad \text{between } m_2 \text{ and } M.$$

In addition, we have the work of the applied forces

$$\begin{matrix} m_1 g \sin\alpha\cdot\delta\xi_1, & & -\,m_2 g \sin\beta\cdot\delta\xi_2, \\ -\,m_1 g\cos\alpha\cdot\delta\eta_1, & \text{and} & -\,m_2 g\cos\beta\cdot\delta\eta_2. \end{matrix}$$

The equations of motion must contain the reactions of the above constraints and the applied forces so that we have

$$\left.\begin{aligned} m_1\ddot{\xi}_1 &= m_1 g\sin\alpha - \lambda_1 + \gamma_1, \\ m_1\ddot{\eta}_1 &= \mu_1 - m_1 g\cos\alpha, \\ m_1 k_1^2\ddot{\theta} &= \lambda_1 b - \gamma_1 a, \end{aligned}\right| \quad \text{for } m_1,$$

$$\left.\begin{aligned} m_2\ddot{\xi}_2 &= \lambda_2 - \gamma_2 - m_2 g\sin\beta, \\ m_2\ddot{\eta}_2 &= \mu_2 - m_2 g\cos\beta, \\ m_2 k_2^2\ddot{\phi} &= \gamma_2 c, \end{aligned}\right| \quad \text{for } m_2,$$

$$\left.\begin{aligned} M\ddot{z} &= \lambda_2\cos\beta - \lambda_1\cos\alpha + \gamma_1\cos\alpha - \gamma_2\cos\beta \\ &\qquad + \mu_1\sin\alpha - \mu_2\sin\beta, \\ I_m\ddot{q} &= (\lambda_1 - \lambda_2)r, \end{aligned}\right| \quad \text{for } M \text{ and } I_m.$$

Evidently λ_1 and λ_2 are the tensions S_1 and S_2 on the two sides of the pulley connecting the cylinders m_1 and m_2, γ_1 and μ_1 are the frictional and normal components F_1 and N_1 between m_1 and M, γ_2 and μ_2 are the frictional and normal components F_2 and N_2 between m_2 and M.

From the kinematic relations we can readily show

$$\ddot{\xi}_1 = \frac{a}{a-b}\ddot{s} - \ddot{z}\cos\alpha, \qquad \ddot{\xi}_2 = \ddot{s} - \ddot{z}\cos\beta,$$

$$\ddot{\eta}_1 = -\,\ddot{z}\sin\alpha, \qquad \ddot{\eta}_2 = \ddot{z}\sin\beta,$$

$$\ddot{\theta} = \frac{\ddot{s}}{a-b}, \qquad \ddot{\phi} = \frac{\ddot{s}}{c}, \qquad \ddot{q} = \frac{\ddot{s}}{r}.$$

The equations of motion in terms of the reactions between the parts and the applied forces with the reduced coördinates are

$$\left.\begin{aligned} m_1\left(\left(\frac{a}{a-b}\right)\ddot{s} - \ddot{z}\cos\alpha\right) &= m_1 g\sin\alpha - S_1 + F_1, \\ m_1\ddot{z}\sin\alpha &= m_1 g\cos\alpha - N_1, \\ m_1k_1^2\frac{\ddot{s}}{a-b} &= S_1 b - F_1 a, \end{aligned}\right|\quad \text{for } m_1,$$

$$\left.\begin{aligned} m_2(\ddot{s} - \ddot{z}\cos\beta) &= S_2 - F_2 - m_2 g\sin\beta, \\ m_2\ddot{z}\sin\beta &= N_2 m_2 g\cos\beta \\ m_2k_2^2\frac{\ddot{s}}{c} &= F_2 c, \end{aligned}\right|\quad \text{for } m_2,$$

$$\left.\begin{aligned} M\ddot{z} &= F_1\cos\alpha + N_1\sin\alpha - F_2\cos\beta - N_2\sin\beta \\ &\qquad + S_2\cos\beta - S_1\cos\alpha, \\ I_m\frac{\ddot{s}}{r^2} &= S_1 - S_2, \end{aligned}\right|\quad \begin{matrix}\text{for } M \\ \text{and} \\ I_m.\end{matrix}$$

Eliminating the reactions between the parts, we verify the previous equations for two degrees of freedom.

It is important to note that in the use of indeterminate multipliers with the equations of constraint we are at liberty to make any choice of independent constraint combinations. Thus, for the determination of the tension of the strings on either side of the pulley we could proceed as follows:

The constraint between the cylinders is

$$-\lambda(\delta s_1 - \delta s_2) = 0$$

or

$$-\lambda(\delta\xi_1 - b\delta\theta + \delta z\cos\alpha - \delta\xi_2 - \delta z\cos\beta) = 0,$$

while the constraint between the string and the pulley is

$$-j(\delta s - r\delta q) = 0$$

or

$$-j(\delta\xi_1 - b\delta\theta + \delta z\cos\alpha - r\delta q) = 0.$$

Therefore, the actual tensions are

$$S_1 = \lambda + j \qquad \text{and} \qquad S_2 = \lambda,$$

where $S_1 - S_2 = j$ = the tangential frictional reaction of the pulley or the increment of tension in the driving side of the string.

(*To be continued.*)

Catalytic Dehydrogenation of Isopropyl Alcohol. W. A. LOTT AND W. G. CHRISTIANSEN (*J. Am. Pharmaceutical Asso.*, 1930, **19**, 570–571) have removed hydrogen from isopropyl alcohol and thereby produced acetone by passing the vapor of the alcohol through an iron furnace which is kept at a temperature of 400° C. and is charged with crucible graphite, made from crystalline Ceylon graphite baked with clay and broken to 8-mesh size. The yield is practically quantitative.

J. S. H.

Poisoning by Thallium. Salts of thallium have been used in this country for a decade as a means of poisoning noxious rodents. Possibly they may also injure and kill live stock. Thus sheep have died while grazing on areas which have been treated with thallium-poisoned grain in order to control prairie dogs. JUSTUS C. WARD (*J. Am. Pharmaceutical Asso.*, 1930, **19**, 556–559) reports that the minimum lethal dose of thallium for sheep is approximately 25 milligrams per kilogram of body weight. Thallium may readily be isolated from the liver and kidneys of the sheep; the fourth stomach and the duodenum show irritated areas, the kidneys evidence of degeneration; death apparently is due to respiratory failure. Sublethal doses as low as 8 milligrams per kilogram of body weight produce symptoms of poisoning including alopecia or slipping of the wool. Sheep do not readily eat thallium-poisoned rolled oats.

J. S. H.

COMMITTEE ON SCIENCE AND THE ARTS.

ABSTRACTS OF REPORTS.

TEMPERATURE CONTROL FOR GAS FURNACES.

HALL OF THE INSTITUTE,
PHILADELPHIA, February 6, 1929.

No. 2879.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the Temperature Control for Gas Furnaces invented by Mr. George Machlet, of the American Gas Furnace Company, of Elizabeth, New Jersey, reports as follows:

The purpose of this device is to enable a gas furnace to control automatically its own temperature by utilizing the minute current generated in a thermocouple as the first step in operating the gas and air valves that supply its fuel.

The difficulty of obtaining satisfactory results by the manually operated methods of the prior art, have made it most desirable to devise some method through which a constant temperature could be secured.

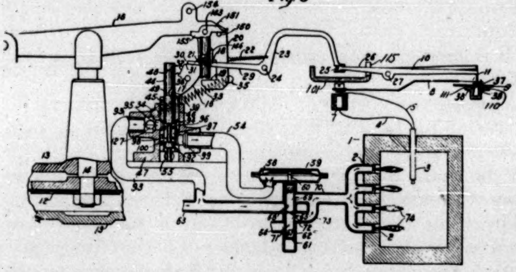
Such a condition is most desirable in the processes of hardening, tempering and annealing for which the gas furnace is especially adapted; so well has the problem been solved by the device under consideration that a temperature can be maintained continuously that will not vary more than five degrees Fahrenheit above or below that for which the device is set.

In general terms, the device may be said to consist of two entirely distinct parts: A pyrometer comprising a thermocouple and a galvanometer, modified in such a way that the pointer of the dial indicates the temperature; and the mechanical means by which the temperature is controlled. This control is secured by making use of the same air pressure that supplies combustion for the gas, through the operation of a pneumatic valve.

In order to describe the details of this device, use is made of a drawing in United States Patent No. 1,033,816, Plate I,

PLATE I

Fig. 5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granted to George Machlet, Jr., on July 30, 1912, on an application filed November 23, 1908. This Plate shows a furnace *1* that is provided with burners *2*, for mixed air and gas into which furnace a thermocouple *3* is inserted.

Wires *4* and *5* connect this to some form of galvanometer with a pivoted rotating coil *7* which carries needle *8* and controls its position. This needle moves along scale *9* and shows the temperature of the furnace.

A prime mover, attached to lever *10*, causes it to vibrate vertically and to bring a projection *11* at its end into, and out of, the path of the moving needle. The purpose of this interference between the needle and lever *10* is to make contact and for this reason the inventor calls lever *10* a "tactor."

A touch of the tactor upon the needle is sufficient to set in motion some form of mechanism by which the feeding of fuel to the furnace is controlled and its temperature regulated.

The especial kind of prime mover suggested in the patent is a compressed-air engine including a diaphragm *12*, inclosed in a casing *13* and connected by rod *14* to a lever arm *15*. Arm *15* is connected to tactor *10* by means of a rocker *16*, pivoted upon the frame work at *17*. Vertical reciprocating motion is secured by means of upper and lower lugs *18* and *19* on an elbow *20* of arm *15*, which engage pin *21*.

Spring *22* tends to force the tactor *10* into the path of the needle but its movement is controlled by the rocker *16*. This spring is attached to a bent arm *23* which is pivoted at *24* on the frame work while at its outer end it is connected by a universal joint *25* to the short arm *26* of the tactor.

Bent arm *23* is extended into an arm *29*, the end *30* of which is pressed against a stop *31* on the rocker *16*, by which the tactor is held up out of the path of the needle.

While the previous description has treated of the position of a tactor to determine the maximum temperature of the furnace, it is to be understood that a similar arrangement is made for a tactor so located as to determine the minimum temperature and thus limit the range of temperature.

The United States Patent on this device has forty-two figures showing its construction in great detail and one hundred ten claims allowed, the forty-eighth reading as follows:

> "The combination with a furnace of an element or device whereby the heat of the furnace produces an electrical current, a volimeter connected to said element and having an indicator to be moved by the current, a tactor, a prime mover yieldingly connected to said tactor to cause the latter to dip repeatedly into the path of said indicator, a primary valve operable by said prime mover only when said tactor is arrested by said indicator, and a secondary valve controlled by the primary valve, and admitting fuel to the furnace; a sear being controlled by the tactor to prevent the movement of the primary valve when the tactor dips into the path of the indicator."

The Institute is of the opinion that the American Gas Furnace Company was among the first to introduce equipment of this kind and that their device has been in continuous use since 1910. Correspondence with users indicates that it is giving satisfactory service at present.

In consideration of his pioneer work in perfecting the automatic temperature control for gas fired furnaces, THE FRANKLIN INSTITUTE awards its CERTIFICATE OF MERIT to MR. GEORGE MACHLET of the American Gas Furnace Company, Elizabeth, New Jersey.

Effect of Heat upon the Digestibility of Milk by Trypsin. ZONJA WALLEN-LAWRENCE AND F. C. KOCH (*Am. J. Diseases of Children*, 1930, **39**, 18–33) find that raw milk, when pasteurized, boiled or evaporated, undergoes changes which render it more readily digested by trypsin (the proteolytic enzyme of the pancreatic juice) *in vitro* than is raw milk. The changes are quantitatively greater for boiled and evaporated milk than for pasteurized milk. This increase in digestibility by trypsin apparently is a function of the temperature to which the milk is heated, and of the length of the period of heating, and is not correlated with a change in either the casein or the buffer system of the milk. It apparently is due to some thermolabile constituent of the milk, probably present in the albumin fraction of the whey.

J. S. H.

Toxicity of Aliphatic Alcohols. WALTER S. EISENMENGER (*Plant Physiology*, 1930, **5**, 131–156) has determined the toxicity of the aliphatic alcohols, from methyl to hexyl inclusive, by measurement of their effect upon the growth of soy bean seedlings. The criteria of growth were the degree of elongation of the root, and the ash content of the shoot. The toxicity, as revealed by the retardation of the growth of the seedlings, apparently did not increase regularly with the number of carbon atoms. The normal alcohol was not always more toxic than its metameric alcohols.

J. S. H.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

CRYSTALLINE RUBBER.

Crude and refined rubber is known to consist principally of hydrocarbons. These hydrocarbons are mixed with smaller quantities of various impurities which can be removed only by a series of long and tedious operations. The highly purified rubber resulting from these operations is a clear, colorless, transparent, elastic solid, as clear and colorless, in fact, as the best plate glass. The problem of ascertaining the composition of this highly purified rubber has heretofore been exceedingly difficult because of the fact that two of the most useful processes ordinarily employed by the chemist, namely, crystallization and distillation, could not be applied to the material. The Bureau of Standards has recently developed a process by which this pure rubber can be repeatedly crystallized, thus opening the possibility of successfully fractionating it into its constituent hydrocarbons and eventually of determining the formulas of their molecules. The crystalline rubber obtained by this process consists of small transparent plates as seen under the microscope. As the crystallization process continues, these plates agglomerate into larger and larger clusters. The crystallization process is carried out by cooling a dilute solution of the highly purified rubber dissolved in ether to a temperature of about 80° below zero Centigrade.

The Bureau has also succeeded in distilling the purified rubber without decomposition by heating it in a high vacuum to the temperature of boiling water. This distilled rubber possesses all of the common characteristics of ordinary rubber, but the rate of distillation is not sufficiently great to make this process of much practical value for purifying the rubber in any quantity. It can, however, be used for obtaining small samples of distilled rubber.

Once having fractionated rubber into its constituent hydrocarbons and determined the formulas of their molecules, the possibility of successfully synthesizing these hydrocarbons is increased at least to the extent that one knows exactly the result to be strived for. Furthermore, there is always the interesting possibility that the pure hydrocarbons may have new and useful properties.

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* Communicated by the Director.

DETERMINATION OF THE EMPIRICAL FORMULA OF A HYDROCARBON.

The precision aspects of the problem of determining the molecular weight and hydrogen content of a hydrocarbon and of combining the results so as to obtain the empirical formula, are considered in a paper in the Bureau of Standards Journal of Research for October. In certain cases the formula can be deduced from the molecular weight alone, in others from the combustion analysis alone. Where both are required the accuracy necessary in one or both is, in many cases, adjustable within rather wide limits and is determinable in any case. A definite laboratory procedure is outlined for obtaining the desired result with the minimum of effort and inconvenience. By following this procedure it should be possible to determine the empirical formula of any pure hydrocarbon containing not more than 100 carbon atoms. A determination of the bromine-addition number may, in some instances, be substituted for the molecular weight determination or for the combustion analysis, or may be utilized to decrease the accuracy which would otherwise be required in either or both of these determinations. The requirements necessary for the determination of a reliable "average formula" of a mixture of hydrocarbons are formulated. The influence of impurities and of polymerization is discussed.

DETERMINATION OF CARBON IN HIGH SULPHUR STEELS BY DIRECT COMBUSTION.

A knowledge of the correct carbon content of steel is necessary to manufacturers, consumers and to all those connected with the steel industry. Perhaps no other element is determined as frequently in steel. The rapid and accurate determination of carbon is therefore a very important problem.

It has been noted that the method used for the determination of carbon in ordinary steels fails when it is applied to high-sulphur steels. The Bureau of Standards has investigated this problem and modified the method so that it can be used for the accurate determination of carbon in steels containing any amount of sulphur. A complete report will appear in the Bureau of Standards Journal of Research for October.

IMPORTANCE OF PARTICLE SIZE IN SAMPLES OF CERTAIN METALLURGICAL MATERIALS.

It is a difficult matter to sample certain metallurgical materials so that their average chemical composition can be determined correctly. Such difficulties arise if the original material is not uniform in composition or if it contains brittle components. The effect of the former is well recognized and can be overcome by sampling the material in a sufficient number of places. The effect of the latter is to cause a concentration of the brittle constituents in the finer portions of the sample, and is often unsuspected. Typical examples are given and corrective measures are suggested in the report which will be published in the October number of the Bureau of Standards Journal of Research.

ANALYSIS OF PHOTOGRAPHIC EMULSIONS.

Investigations of photographic sensitivity by independent methods have all led to the conclusion that in highly sensitive emulsions the grains of silver halide contain traces of some foreign material acting as "sensitivity nuclei"; without specifying the means of their action, these decrease the exposure to light necessary to make the silver halide developable. Since sensitivity may be increased by digesting (ripening) the silver halide with the gelatin of the emulsion at an elevated temperature, it was reasonable to assume that the sensitivity nuclei are metallic silver produced by reduction; this is supported by the marked decrease in sensitivity of ripened emulsions on treatment with oxidizing agents. The Eastman Kodak laboratories have also definitely established that photographically active gelatins contain sulphur compounds which react with silver bromide, under the conditions of emulsion making, to form silver sulphide; and that this leads to an increase in sensitivity similar to that produced by digestion. Chemical analysis of the sensitivity nuclei would be highly desirable, but the optimum amount of sensitizing sulphur compounds is so small that this appeared almost impossible.

Recently, however, two laboratories have independently reported the presence in unexposed photographic emulsions of silver, or a compound of silver other than a halide. Weigert and Lühr [1] fixed finished plates in slightly alkaline thiosulphate solutions, washed thoroughly, repeated the thiosulphate bath and washing, then dissolved the silver remaining in the emulsion film by nitric acid, and analyzed by potentiometric titration. From 0.01 to 0.04 per cent. of the total silver remained in the emulsion after this treatment. This is an unexpectedly large amount, since direct photochemical decomposition of silver halides to this extent may produce visible darkening. Experimental emulsions gave results of the same order, with the amount of silver increasing on digestion as would be predicted. If the silver found by this method is originally present in the emulsion it must be metallic silver or silver sulphide. Weigert considers it is the true photosensitive component of the emulsion, rather than the silver halide.

Schmidt and Pretschner [2] digested large quantities of very thoroughly washed (uncoated) emulsion with nitric acid, and made a gravimetric analysis for silver in the solution with elaborate precautions. They report that in fast emulsions, containing about 97 per cent. AgBr and 3 per cent. AgI, the silver is 0.08 per cent. in excess of the halogens; in silver chloride emulsions the excess may be as much as 0.6 per cent. The excess silver found by their method varies directly with the solubility of the silver halide and is independent of the speed of the emulsion. It therefore seems impossible that there can be a direct connection between these determinations and the sensitivity nuclei. Schmidt and Pretschner further found that, on centrifuging the silver halide grains out of an emulsion, most of the excess silver was found in the gelatin. They conclude that in silver chloride emulsions a silver-gelatin compound may be present, but that this compound is decomposed by bromide or iodide and cannot exist in silver bromide emulsions. Silver in such a compound would be determined by their method, but not by that of Weigert and Lühr.

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[1] Weigert and Lühr, *Naturwissenchaften*, **15**, 788 (1927); *Zeit. f. Elektrochemie*, **34**, 605–10 (1928); *Zeit. f. wiss. Phot.*, **27**, 283–303, 312–37 (1930).

[2] Schmidt and Pretschner, *Zeit. f. wiss. Phot.*, **25**, 293–307, 354–62 (1928); **26**, 86–95, 259–74, 375–80 (1928–29); **27**, 36–47, 173–77 (1929); **28**, 30–4, 35–40, 111–17 (1930).

The Bureau of Standards in studying the effects of excess silver or halide in photographic emulsions has repeated some of the work of the German investigators; potentiometric titration has been substituted for the gravimetric methods used by Schmidt and Pretschner. It is found that the combination of silver ion with gelatin is much stronger than had been expected and can account for the large values obtained by Schmidt and Pretschner. It is dependent on the hydrogen concentration; in an emulsion ready for coating, the free silver ion concentration rose from 2×10^{-7} N to 7×10^{-6} N when the pH was changed from 7 to 4. On extremely thorough washing, both of fast commercial plates with 4 per cent. AgI and of experimental emulsions of similar composition but made so as to contain the minimum of ripening nuclei, the excess silver as determined by nitric acid extraction rose to an equilibrium value of 0.3 to 0.4 per cent.; the immediate change in photographic properties was very small. This excess cannot be metallic silver for at least 90 per cent. of it can be extracted by N/100 acetic acid; similarly, if the emulsion is stripped with hot water and acidified to a pH less than 4.7, there is a sudden rise in silver ion concentration, and electrometric titration gives a value almost identical with nitric acid extraction. In an emulsion containing the normal small amounts of soluble bromide, the silver ion concentration is so reduced that the formation of silver gelatinate is inappreciable, but on prolonged washing, with the bromide ion concentration kept at a minimum, the silver ion activity is so increased that relatively large amounts are formed. Data on the equilibrium between silver and hydrogen ions and gelatin will be given in the complete report of this work in the Bureau of Standards Journal of Research.

Results with the thiosulphate method of analysis were in general agreement with those reported by Weigert and Lühr, although tests with known suspensions of colloidal silver in gelatin indicate that there is probably an appreciable loss of silver oxidized by dissolved oxygen in the solutions. The analytical agreement does not in itself confirm the views of Weigert as to the photographic importance of this silver. It is well known that considerable amounts of colloidal silver may be introduced into an emulsion under certain conditions with very little photographic effect, and it seems reasonable that most of the silver formed during the preparation of the emulsion should be dispersed through the gelatin and not associated with the silver halide grains as sensitivity nuclei. The analytical determination of the nuclei, if they can be isolated, will still be a matter of very great difficulty.

THE MEASUREMENT OF SOUND ABSORPTION.

The Bureau of Standards Journal of Research for October will contain an article giving a description of the different methods used for the measurement of sound absorption coefficients.

The theory is given rather briefly and the requirements are described which must be met if satisfactory measurements of the sound absorption coefficients are to be obtained.

A description and sketch of the new reverberation room are given. The requirements of a satisfactory source of sound are discussed and graphs are given showing that the loud speaker, which was used as the source of sound, met these requirements.

The method of calibrating the room and determining the sound absorption coefficients by using the ear as a measuring instrument is also described. As the observer's judgment modifies these measurements to some extent it is necessary to take a large number of measurements. Several thousand measurements were required to calibrate the room and about 1000 measurements more were necessary to determine the sound absorption coefficients of a sample for 6 frequencies.

This required too much time and work, and attempts were therefore made to change to an instrumental method of some kind which would be more accurate and also shorten the time required.

The first attempt was to use an oscillograph for obtaining a photograph of the decay of the sound. It was possible to compute the absorption from the information on the photograph but the amount of work involved proved to be excessive.

The method in use at the present time is a purely instrumental one. The rate of decay is determined by the use of a timer and a vacuum tube voltmeter. The results obtained in this way agree well with those obtained by the reverberation method.

NEW ZEALAND FLAX AS A PAPER-MAKING MATERIAL.

An investigation to determine the paper-making value of New Zealand flax has been in progress. Samples of the flax—unscutched, scutched, and tow—which at present is used commercially only for cordage, were submitted to the bureau with request for information relative to the suitability of the material for the production of paper.

Paper-making tests have been made on both laboratory and semi-commercial scale. The test procedure followed was essentially the same as that generally observed in the production of paper.

Cooks were made using caustic soda, caustic soda and sodium sulphide combined, and sodium sulphite and caustic soda separately, as the reducing agents.

When the flax was cooked by the caustic soda process with comparatively small amounts of caustic (10 per cent.) the pulp was suitable for good bag or wrapping paper; with larger amounts of caustic (25 per cent.) it could be bleached with about 14 to 16 per cent. of bleach and was then satisfactory to use in admixture with other pulps in making fine papers.

The yield of air-dried unbleached pulp from the 25 per cent. caustic cook of unscutched New Zealand flax was 57.2 per cent., based on the bone-dry weight of the flax, in the semicommercial tests, and 57.7 per cent., in the laboratory test. The results of the laboratory and semicommercial tests therefore checked very closely as to yield when similar cooking conditions and amounts of chemical were employed. When 10 per cent. of caustic soda was used the yield of unbleached pulp was 77.4 per cent. This pulp was suited only for wrapping paper, however, whereas that from the 25 per cent. caustic soda cook could be bleached and the resultant pulp was satisfactory for use in fine paper.

Unscutched flax cooked with 15 per cent. of caustic soda and 10 per cent. of sodium sulphide combined gave a yield of 62.2 per cent. The pulp obtained was satisfactory for use only in wrapping paper.

The pulp from the two-stage cooks—the first with 25 per cent. of sodium sulphite, the second with 10 per cent. of caustic soda—seemed to give the most satisfactory results. The yields of unbleached pulp were: 37.2 per cent. for unscutched flax; 48.5 per cent. for the scutched flax when not dusted; 60–65 per cent. for the dusted scutched flax; and 56.9 per cent. for the dusted tow.

Some of the test results on the machine-made papers are given in the following table on page 517.

The results of the tests thus far seem to indicate that if the flax can be properly cleaned, freed from dead fibrous materials, and the woody sections eliminated from the scutched material, a fair grade of pulp suitable for use with other pulps in making a fine paper can be obtained.

As soon as the investigation is completed a report of the work will be prepared for publication.

FIRE TESTS OF WELDED STEEL FLOOR CONSTRUCTION.

A research project has been undertaken in coöperation with the American Institute of Steel Construction to determine the fire resistance of a new type of welded steel floor construction. The structural elements of this type of floor consist of rolled steel beams, generally 3-inch, 4-inch or 5-inch in depth, spanning between girders, the upper flanges of the beams being welded to steel plates generally ¼-inch thick. The beams are spaced 2 feet apart, the steel plate serving the purpose of the usual floor slab.

Fire exposure will be applied to the floors from below in most tests but in six tests fire exposure from above will be applied by burning combustible materials such as wood and paper in amounts from 10 to 40 pounds per square foot of

	Weight 25 × 40, 500 sheets.	Bursting strength.	Folding Endurance.		Tensile Strength.		Tearing Strength.	
			Machine Direction.	Cross Direction.	Machine Direction.	Cross Direction.	Machine Direction.	Cross Direction.
	pounds	lbs. in.	double folds		kilograms		grams	
Flax—unscutched, bleached 25 per cent. caustic soda cook	56.8	37.7	341	487	9.1	6.2	64.4	63.4
Flax—unscutched, unbleached; 25 per cent. caustic soda cook	58.9	61.6	4,060	2,824	11.9	7.9	106.6	98.4
Flax—unscutched, unbleached; 10 per cent. caustic soda cook	57.5	52.5	1,510	2,250	10.8	6.5	114.0	95.2
Flax—scutched, bleached; 2-stage cook, 25 per Na_2SO_3 and 10 per cent. NaOH	56.1	40.1	670	[illegible]305	9.2	6.0	63.8	67.6
Flax—tow, bleached; 25 per cent. caustic soda cook	55.1	42.6	700	735	9.3	5.8	71.2	78.0

floor area. Various degrees of protection will be given the upper surface of the steel plate floor for these tests, from the bare plate to protections of 2-inch thickness of light-weight concrete.

The fire exposure from below will be obtained with a gas furnace using 48 horizontal venturi-tube induction burners. The protection applied to the beams will vary for the different tests from a ¾-inch thickness of plaster on metal lath to 2-inch gypsum slabs hung from the lower beam flanges to form a suspended ceiling. Three tests in which a hose stream will be applied to the heated floor after a period of fire exposure will also be included.

The floors will be supported during test within a heavy steel restraining frame, and superimposed loads of 45 to 160 pounds per square foot will be maintained constant during the fire tests. For some tests the floor connections with the restraining frame will be designed to give full restraint to the construction during tests and for others the ends of the beams and plates will be freely supported with opportunity for expansion. The test floors will be 18 feet long and 13½ feet wide. The beams will span the full 18 foot length of the floor except in five tests where the beams will be supported on a girder at an intermediate point to introduce the details of fireproofing required for the girder.

A STUDY OF WEATHER ACTION ON MARBLE.

A large block of large crystal calcite marble recently obtained from near the top of a chimney on the U. S. Patent Office Building where it had been exposed to severe weather conditions for about seventy years afforded a good opportunity for studying how the weather attacks this building material. The dimension between the face exposed on the outside of chimney and the inner face was 8 inches. The outer face was freely exposed to normal weathering conditions while the inner face was protected to a small degree by the chimney walls but exposed to any chemical actions resulting from the collection of soot.

This block was sliced into slabs parallel to the weathered faces and tested for strength and absorption. Strength determinations showed that the marble at the outside face had lost about 70 per cent. of its original strength and at the inside face about 60 per cent. The strength increased uniformly toward the middle to about 55 per cent. of the original. Absorption results indicated that the porosity of the material had increased to nearly three times the original at the weathered faces but only about 40 per cent. at the middle.

Specimens cut from the weathered faces were subjected to freezing and thawing tests under the most severe conditions which could be devised. Although these have undergone more than 700 cycles of this test no appreciable amount of decay has been noted. Many well known building materials would be totally disintegrated by such a test but the indications are that frost has very little effect on marble.

Appearance of the exposed surface as well as strength and absorption tests show that this marble has been altered from its original state. Many parts of the marble in this building show crystals entirely loosened and the surface crumbling away.

It has long been recognized that the acid condition of rainwater causes an appreciable amount of surface solution on carbonate rocks but it seems to have been assumed that the acid is neutralized before it penetrates the pores. This study affords considerable evidence that acid action takes place within the pores as well as at the surface, resulting in a gradual enlargement of the pores until the crystals are entirely loosened.

X-RAY STUDY OF CONSTITUTION OF PORTLAND CEMENT.

The X-ray diffraction method has been applied to a study of the constitution of Portland cement clinker.

It has been found that $2CaO.SiO_2$ and CaO combine to form $3CaO.SiO_2$ and not a solid solution of $2CaO.SiO_2 + CaO$; that $8CaO.Al_2O_3.2SiO_2$ does not exist in Portland cement systems; that solid solutions are not formed between the aluminates and silicates of cement systems; and that free CaO is not present generally in commercial cements in amounts as great as 2.5 per cent.

The limiting amounts were determined of each cement compound that could be identified by X-ray means as employed in this study. Twenty-eight commercial cement clinkers were subjected to X-ray examination and $3CaO.SiO_2$ and $62CaO.SiO_2$ were identified in each. $3CaO.Al_2O_3$, $4CaO.$-$Al_2O_3.Fe_2O_3$ and MgO were found in the samples which contained those compounds, by calculation, in amounts great enough to permit of identification.

A full report will be made in the October number of the Bureau of Standards Journal of Research.

FURTHER STUDIES OF THE "MOISTURE EXPANSION" OF WHITEWARE.

An endeavor is being made to determine the nature of the reaction manifested by an increase in volume when water permeable ceramic bodies are subjected to moisture. The magnitude of these changes, and their relation to body composition and degree of water permeability, has been presented in Technical News Bulletins Nos. 146, 153, 156, and 160 (June, 1929, and January, April, and August, 1930). The minimum temperature at which the moisture causing this expansion can be removed from the body, and the accompanying contraction completed, should indicate whether or not actual hydration has taken place. This minimum temperature was determined approximately by holding previously autoclaved water permeable specimens at specific temperatures and observing the shrinkage by means of the interferometer, the specimens being weighed before and after these heat treatments to determine the change in weight. Specimens of three commercial bodies were tested. Although the specimens were dried to constant weight at 110° C. before the autoclave treatment, the loss in weight when held at temperatures of 120° and higher exceeded the gain in the autoclave. The gain in the autoclave varied from about 10 to 15 per cent., while the weights after the heat treatments were from 0.0 to 0.7 per cent. less than the original weights after drying at 110° C. The specimens continued to lose weight up to approximately 250°. That this temperature approximates the minimum required to counteract moisture expansion is also indicated by the contraction in volume. Specimens held at approximately 120, 180, and 200° until contraction practically ceased showed further contraction at higher temperatures, but when held at 240–250° until contraction ceased showed no further evidence of contraction, due to elimination of moisture, at higher temperatures. The greatest contraction noted, due to elimination of moisture, was obtained by holding a specimen at 240–250° C. for five hours and equalled 0.075 per cent. That the moisture causing expansion cannot be eliminated at temperatures below approximately 250° C. indicates that it is in more intimate contact than so-called pore water and hygroscopic moisture. Just what the nature of the reaction or combination between the moisture and the specimens is, has not been demonstrated.

Atomic Weight of Vanadium. ARTHUR F. SCOTT AND CLYDE R. JOHNSON (*Journal of the Am. Chem. Soc.*, 1930, **52**, 2638–2648) have determined the atomic weight of vanadium by measurement of the ratio of vanadyl trichloride to silver. The new value of this atomic weight is found to be 50.947.

J. S. H.

Sodium Pyroantimoniate. WILLIAM F. POND (*Chemist Analyst*, 1930, **19**, No. 2, 11–14) has studied the quantitative determination of sodium as its pyroantimoniate. Salts of lead, bismuth, copper, cadmium, antimony, iron, aluminium, chromium, manganese, zinc. cobalt, nickel, barium, stronthium, and calcium, and free hydrochloric acid, nitric acid, and sulphuric acid must be absent. The reagent is a 2.5 per cent. solution of sodium-free potassium pyroantimoniate; the salt is dissolved in water which is almost boiling; the solution is cooled in ice water, mixed with 30 cc. of a 10 per cent. solution of sodium-free potassium hydroxide, filtered, and stored in tightly stoppered bottles. The reagent keeps for 5 or 6 weeks. The sample (0.2 to 0.25 grain) is dissolved in 100 cc. of water acidified with hydrochloric acid. The solution is boiled, rendered slightly alkaline by addition of potassium hydroxide, cooled, treated with an excess of the potassium antimoniate solution (approximately 100 cc.), stirred with a rubber policeman for 5 minutes, and let stand for at least 3 hours and, preferably, over night. The precipitated sodium pyroantimoniate is crystalline, is collected in a tared Gooch crucible, washed with water, dried for 3 hours at a temperature of 210 to 212° F. (98.9 to 100° C.), cooled in a desiccator, and weighed.

J. S. H.

THE FRANKLIN INSTITUTE.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, September 17, 1930.*)

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BOOK REVIEWS.

STRENGTH OF MATERIALS, by S. Timoshenko. Part I, Elementary theory and problems; Part II, advanced theory and problems. 2 volumes, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1930. Price $8.00.

The gap between the ordinary and more elementary treatises on the Strength of Materials and advanced works in the Theory of Elasticity is admirably supplied in this new treatise. Though the first volume is an elementary exposition primarily on the Strength of Materials, the method of attack is particularly commendable as a foundation for the more advanced work covered in the second volume. The author manifests the unique ability of maintaining an excellent analytical exposition of the subject while at the same time focussing on the application of the subject to actual and important engineering problems of a distinct practical nature. In fact, the combination of an engineering aspect together with the necessary analytical foundation is an additional feature of this work.

A review of some of the important treatises on this subject is of interest in noting the particular value of this contribution to the literature of the subject.

We find treatises on the Strength of Materials and the Theory of Elasticity quite definitely divided into a distinct engineering treatment on the one hand and a highly mathematical treatment as a branch of applied physics on the other. As typical and excellent engineering treatises, we note works on the order of Morley's "Strength of Materials," Bach's "Elasticity," etc. For excellent mathematical treatises, we note the celebrated works of Love, Clebsch, etc. Treatments of an in-between character are not numerous. Perhaps one of the best expositions is found in the works of Föppl and particularly the latter work on "Drang and Drung" and the author's own exposition of the subject in the Russian language. On the other hand in this domain we find many valuable papers on particular subjects and problems, as in the works of Castigliano, Southwell, Lamb, Stodolo, Prandtl, etc., as well as in publications by the author himself. Advanced and intermediate theorems and applications of applied elasticity are included in the treatment of other subjects as in Raleigh's "Theory of Sound," where the original introduction of trigonometrical series, treated as generalized coördinates, in the expressions for deflections for bars, plates, etc., as well as generalizations of Maxwells' reciprocal relations are to be found. Also in this domain must be included Thomson and Taits' second volume on "Natural Philosophy." On the other hand we find considerations on special elastic problems but more of an engineering aspect in Stodolo's "Steam Turbines," Bach's "Maschinen Elemente," Boulvin's "Organes des Machines." Finally we must mention the great work of Castigliano in his treatise on "Elastic Structures," where the conception of the introduction of the redundant reactions and external loads as coördinates in the elastic potential of indeterminate structures was first introduced. The history of the subject with many interesting problems is found in Todhunter & Pearson's "History of Elasticity."

The author has not only coördinated this in-between field compactly in two volumes, but must be given further credit for additional material and the extension of important problems and simplified presentations particularly applicable for advanced engineering work.

Conspicuous and original contributions are to be found in the first volume, in the author's treatment of the deflections of beams, in the introduction of plastic and non-linear bending, in combined bending and torsion with consideration of the crank shaft problem and in his welcome treatment of Energy of Strain, a simple introduction of Castigliano's theorem to statically indeterminate analysis and particularly a clear presentation of the reciprocal theorem that is not to be found in the usual engineering treatises. The second volume includes more advanced problems such as the bending of bars on elastic foundations, its extension to the analysis of tanks, the application of trigonometric series for deflections, a very satisfactory treatment of curved bars with interesting applications, together with the author's original contribution on stress concentration around holes. We find also included the theory of bending in plates, stresses in tanks and the equations of the rotating disk. The author includes original contributions to elastic stability with the buckling of bars, plates and shells. The treatment of stress concentration, a matter to which the author has given considerable study, should prove of considerable use in engineering design. The final chapter is a treatment of the properties of material and is of value due to its coördination with the previous parts of the work, blending the statistical knowledge of the subject with an analytical background so that proper interpretation of stress analysis can be made in the design of engineering structures.

On the whole this work must be rated as an original and valuable exposition of the subject and undoubtedly one of the best treatises on the Strength of Materials.

R. EKSERGIAN.

ALLEN'S COMMERCIAL ORGANIC ANALYSIS, Volume 8, Fifth Edition, Revised and partly rewritten. Edited by C. Ainsworth Mitchell, M.A., D.Sc., F.I.C. Assisted by collaborators. 8vo, 761 pages, Philadelphia, P. Blakiston's Son & Co., Inc. Price $7.50.

Under the modest title of "An Introduction to the Practice of Commercial Organic Analysis," was published in England a little over half a century ago the initial volume of 360 pages, including indexes, dealing with many problems which had lately arisen to puzzle commercial analysts and works-chemists. Organic chemistry had begun and indeed fairly well entered upon its active development, and many problems were presenting themselves in connection with merchandise of all kinds and manufacturing operations. Germany had succeeded by skillful coöperation of theory and practice in getting control of the dyestuff industries, and these colors had by that time begun to replace those derived from natural sources and to enter not only into the ordinary operations of textile dyeing, but to the coloring of various articles used as food or accessories at the table. The first volume of this initial printing was devoted, as the note on the title page states to "Cyanogen Compounds, Alcohols and Their Derivatives, Phenols, Acids, &c." The second volume covering other types of compounds followed soon. The distinguished author, who contributed during his long active life many important tests and methods, continued his work of collecting data, and later editions were of greater scope and of larger volumes. After his health gave way so as to prevent him from taking entire charge of the revision of his book, the present reviewer assumed partial charge, and an incomplete revision of a portion of the book was issued hastily, to meet an attempted piracy. Later a more formal arrangement was made, and an edition of certain volumes of the book was printed in this country. Conditions are now completely changed. The author has passed away. The extent of the material now necessary to be considered in such a work has so increased that a number of experts and specialists have to be taken into assistance in the preparation of the volumes.

The eighth volume just published was intended to include those departments of chemistry that are more directly related to biology and physiology, but even in this field the material has been too great to be placed within the compass of a convenient book. Hence the ninth volume now in preparation will cover the more distinctly biochemistry field and it is to be noted that in this volume a section on toxicology will be included.

Concerning the present work it needs only to be said that it contains a very large amount of valuable matter contributed by specialists in Great Britain and the United States, the latter being represented, by sections prepared by four distinguished American Biochemists. The book, of course, will be needed in every active laboratory, and will maintain the reputation which C.O.A. has enjoyed for half a century.

HENRY LEFFMANN.

LECONS SUR LES CONDUITES, par M. Charles Camichel, Correspondant de l'Academie des Sciences, Professeur a l'Universite de Toulouse, VI—101 pages, 8vo, paper. Paris. Gauthier-Villars et Cie., 1930. Price 30 francs.

The pipe lines of hydraulic power installations present problems which are of comparatively little consequence in the current practice of design of lines for domestic and industrial water supply where considerations of steady flow suffice. The high transient pressures in power lines incident to sudden variations of flow entail a searching study into the resulting "hydraulic ram" effects which they cause in such lines. This work deals with a rational analysis of the flow of water in pipes with especial reference to the transient pressures to which they are subjected as a function of time. The analysis is based upon a course of lectures by the author in 1926 at the University of Paris on the Mechanics of Fluids.

The first and major division of the two parts of the volume treats of the phenomena of flow in pipes of constant and variable diameter. General equations are derived for pressures and velocities at a given instant as well as the formulas of Alliéve for the velocity and propagation of a wave. The analysis of the phenomena arising from the sudden or gradual closing of the line with its resulting hydraulic ram effect, the examination of the resulting loss of head and the phenomena of resonance are of equal interest and importance. A study of the application of artificial air pockets for the mitigation of shocks according to the methods of Rateau is of obvious value in the actual design of a pipe line. The second part treats of the arrangement of lines; placement and anchorages.

The work is a highly rational analysis of the flow of water in pipes and a valuable contribution to sound methods in the design of hydraulic pipe lines.

LUCIEN E. PICOLET.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 348. Strength of Welded Joints in Tubular Members for Aircraft. By H. L. Whittemore and W. C. Brueggeman. 41 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price thirty cents.

This investigation was made by the Bureau of Standards in coöperation with the National Advisory Committee for Aeronautics for the Aeronautics Branch of the Department of Commerce. The object of the investigation is to make available to the aircraft industry authoritative information on the strength, weight, and cost of a number of types of welded joints. This information will, also, assist the Aeronautics Branch in its work of licensing aircraft by providing data from which the strength of a given joint may be estimated. As very little material on the strength of aircraft welds has been published, it is believed that such tests made by a disinterested governmental laboratory should be of considerable value to the aircraft industry.

Following the program prepared from information supplied by manufacturers, 40 joints were welded under procedure specifications and tested to determine their strengths. The weight and time required to fabricate were also measured for each joint.

Report No. 349. A Proof of the Theorem Regarding the Distribution of Lift Over the Span, for Minimum Induced Drag. By W. F. Durand. 15 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

The proof of the theorem that the elliptical distribution of lift over the span is that which will give rise to the minimum induced drag has been given in a variety of ways, generally speaking too difficult to be readily followed by the graduate of the average good technical school of the present day. In the form of proof herewith, an effort is made to bring the matter more readily within the grasp of this class of readers. The steps in the proof, briefly outlined, are as follows:

1. Given a basic distribution of lift across the span denoted by (*a*) with a second supplementary distribution denoted by (1). Then it is shown that the induced drag of lift (*a*) in the downwash due to lift (1) and the induced drag of lift (1) in the downwash due to lift (*a*) are equal, and that the total effect of the small distribution (1) on the induced drag will be measured by twice either of these small quantities.

2. Next two small changes are assumed in a basic distribution (*a*). These are represented by (1) and (2), and are further assumed to be equal in amount and opposite in algebraic sign, thus leaving the original lift unchanged in amount, but changed in distribution. Under these conditions it is then shown that in order for the distribution (*a*) to be that for minimum induced drag, the change in induced drag due to this small change in distribution must be zero.

3. It is next shown that for any pair of small changes such as (1) and (2) the only value of the basic downwash which will meet the condition of step (2) is downwash constant across the span.

4. It is known mathematically that the elliptical distribution across the span is that which gives a constant value of the downwash and hence as a result of (1), (2), (3), this must be the distribution which will give the minimum value of the induced drag.

Report No. 353. Airfoil Pressure Distribution Investigation in the Variable Density Wind Tunnel. By Eastman N. Jacobs, John Stack, and Robert M. Pinkerton. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

With a view to extending the knowledge of the aerodynamics of lifting surfaces, the distribution of pressure over one section each of six airfoils has been measured in the Variable Density Wind Tunnel of the National Advisory Committee for Aeronautics. The following airfoils were investigated: N. A. C. A. 85-J, N. A. C. A. 84-J, N. A. C. A. 84, N. A. C. A. M-6, Clark Y, and R. A. F. 30.

Pressure-distribution diagrams, as well as the integrated characteristics of the airfoils, are given for both a high and a low dynamic scale Reynolds Number or Vl/ν, for comparison with flight and other wind-tunnel tests, respectively. It is concluded that the scale effect is very important only at angles of attack near the burble. The distribution of pressure over an airfoil having a Joukowski section is compared with the theoretically derived distribution. A further study of the distribution of pressure over all of the airfoils resulted in the development of an approximate method of predicting the pressure distribution along the chord of any normal airfoil for all attitudes within the working range if the distribution at one attitude is known.

Report No. 357. Aircraft Accidents—Method of Analysis. Report prepared by Committee on Aircraft Accidents. 17 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

This report is a revision and expansion of the report prepared by the Special Committee on the Nomenclature, Subdivision, and Classification of Aircraft Accidents, which was issued as Technical Report No. 308 of the National Advisory Committee for Aeronautics. The special committee was organized by the National Advisory Committee for Aeronautics in response to a request dated February 18, 1928, from the Air Coördination Committee, which consists of the Assistant Secretaries for Aeronautics in the Department of War, Navy, and Commerce.

In presenting this report for publication the Committee on Aircraft Accidents appreciates that it can not represent a completed project, as the study of aircraft accidents can never be finished. The report represents the experience of the committee and of the three departments concerned in the study of aircraft accidents up to the present time. The study of aircraft accidents for the purpose of analyzing them in such a manner as to assist in reducing their frequency and severity is a task which can never be completed, but must be continued in step with the progress of the art.

HUBERT HERMANNS TECHNO-DIKTIONAR, DEUTSCH—ENGLISCH—ITALIENISCH, 432 pages, 16mo. Berlin, Hubert Hermanns, 1929. Price $3.75.

The "confusion of tongues" has become a serious drawback to the diffusion of scientific knowledge. If we note the history of scientific development in the earlier centuries since the revival of learning, we note the free interchange of information between scientists of the more important European nations, and also personal associations that seem to have been quite satisfactory. This was due largely to the fact that educated men were fairly familiar with Latin. Correspondence and exchange of papers were much facilitated. Even conversation was possible between men of different languages, through the common tongue. Latin, however, is now in the discard, and international gatherings are faced with the question of designating a few official languages and excluding all others, sometimes much to the inconvenience of competent delegates.

For many years German and French have sufficed fairly well for international tongues, but several of the nations of Europe have of late years developed activities in science, especially in the applied phases and some familiarity with these is necessary. Italian is one of the most important of these. Its now extensive literature in technical fields makes some knowledge of it essential to the engineer. The pocket volume in hand is an opportune publication. It contains a large amount of information on subjects of importance and special application.

HENRY LEFFMANN.

PUBLICATIONS RECEIVED.

Allen's Commercial Organic Analysis, a treatise on the properties, modes of analysis, and approximate analytical examination of the various organic chemicals and products employed in the arts, manufactures, medicine, etc. Vol. VIII. Glucosides, Non-Glucosidal Bitter Principles, Enzymes, Putrefaction bases, Animal bases, Animal Acids, The Cyanogen Compounds, The Proteins, The Digestion Products of Proteins. Editor, C. Ainsworth Mitchell. Fifth edition, revised and partly rewritten. 761 pages, illustrations, 8vo. Philadelphia, P. Blakiston's Son & Company, 1930. Price $7.50.

Hubert Hermanns Techno-Diktionar, Deutsch—Englisch—Italienisch, 432 pages, 16mo. Berlin, Hubert Hermanns, 1919. Price $3.75.

La Mecanique Ondulatoire et les Nouvelles Theories Quantiques, par Arthur Haas, 198 pages, 12mo. Paris, Gauthier-Villars et Cie, 1930. Price 30 frs.

The Place of Engineering Science in University Studies. An inaugural lecture delivered before the University of Oxford on 7 June 1930 by R. V. Southwell, professor of Engineering Science. 24 pages, 8vo. New York, Oxford University Press, 1930. Price 70 cents.

Science and the Scientific Mind, by Leo E. Saidla and Warren E. Gibbs, first edition, 506 pages, 8vo. New York, McGraw-Hill Book Company, Inc., 1930. Price $3.00.

The Theory of Electrical Artificial Lines and Filters, by A. C. Bartlett, B.A. 155 pages, 8vo. New York, John Wiley & Sons, Inc., 1930. Price $3.50.

CURRENT TOPICS.

The Nature of the Penetrating Radiation. W. Bothe and W. Kohlhoerster. (*Zeit. f. Physik*, Vol. 56, p. 751.) "The investigation of the penetrating radiation has followed a peculiar course in this that the most different properties of the radiation, its intensity distribution, absorption, scattering and even its origin have been examined and discussed, while the really fundamental question of its nature has hitherto found no experimental answer. If hitherto by common agreement preference has been given to the view that the penetrating radiation is a very hard γ-radiation, this came to pass solely on account of its enormous penetrating power which only with greater difficulty could be thought to be associated with a corporeal radiation. Always the fundamental possibility of electron radiation has been pointed out by nearly all the authors concerned." Such a stream of electrons has been sought by Schweidler, Swann and Hoffmann in order to explain the maintenance of the negative charge of the earth. They endeavored to find an accumulation of electric charge on an insulated metallic conductor as an effect of the hypothetical stream of electrons, but their results were negative. On the other hand, Skobelzyn, using C. T. R. Wilson's cloud apparatus has demonstrated the presence of electrons moving with high speed and not of radioactive origin.

In order to investigate the nature of the penetrating radiation Geiger and Müller's tubes for counting electrons were used. Two tubes with their axes parallel were placed a few centimeters apart. The arrival of an electron in each tube caused a deflection of the attached electrometer. The deflections due to both tubes were registered on the same film. Altogether about 90,000 such deflections were recorded. When deflections were registered at the same instant from both tubes this coincidence was interpreted to mean that the same particle had traversed both tubes. Experiments were conducted both in the basement and in the attic of the main building of the Reichsanstalt in Charlottenburg. As an example, one series in the basement lasted 15 minutes. On the 823 cm. of film used there were 619 deflections due to the upper tube and 364 to the lower, which was vertically beneath the other. Of these 33 coincided in time of occurrence. The number of accidental coincidences was calculated to be 5.5. This leaves 27.5 actual coincidences. When a block of gold was placed between the two tubes it caused a less difference than might have been anticipated. Without the gold and for one minute the deflections from the upper tube were 40.3, from the lower tube, 23.6 and of coincidences 1.67. With the gold in position the corresponding values were 39.8, 22.4 and 1.65. Similar experiments were conducted in the attic and metal screens were removed from the apparatus. Here, without the gold the upper deflections, lower deflections and coincidences per minute were 55.3, 33.9 and 2.74 and with the gold block they were respectively 55.0, 29.3 and 2.06. To reach the basement the radiation had to penetrate several thick cement floors and the metal sheathing of the instrument, all of which were equivalent to a layer of water 3 meters thick. The rays or particles that finally reached the basement had great penetrating power and hence were little affected by the introduction of the gold.

The effect of varying the vertical distance between the two tubes was examined. The coincidences found in the attic being expressed as percentages of the deflections due to the lower tube, for a distance between the tube axes of 54 mm. the percentage was 22.5, for 74 mm., 14.6 and for 104 mm., 9.7. When the tubes were placed side by side instead of one above the other the percentages dropped to about half of the former value. This shows that the radiation is not equally distributed in all directions but is strongest in a vertical direction. This peculiarity of distribution of intensity has been observed to hold for the penetrating radiation.

The authors now discusses the possibility of explaining the observed facts by supposing that electrons are set free by a γ-ray and that one such electron goes to one tube and a second to another. The probabilities, it appears, are strongly against this and the conclusion is reached that "each coincidence must signify the passage of one and the same beam of corpuscles through the tubes." Further examination shows that "the beams of corpuscles are just as strongly absorbed as is the penetrating radiation under the same conditions." A vain attempt is made to interpret the corpuscular radiation as caused by the action of γ-rays as hard as it is. "Though the question of the origin of the penetrating radiation receives no answer through our results, yet there is need of a thorough investigation of all the facts in the case. As long as it is believed that the radiation is of the nature of γ-rays, we are almost compelled to think of processes of atomic formation." A corpuscular radiation could, however, derive its energy by other means. G. F. S.

The Effect of Pressure on the Rigidity of Steel and Several Varieties of Glass. P. W. BRIDGMAN. (*Proc. Am. Acad. Arts and*

Sci., Vol. 63, No. 10.) The author begins with the surprising statement "The change of rigidity or shearing modulus under pressure seems to have never been determined." So much hard work and scientific acumen have been devoted to the measurement of all sorts of quantities under all sorts of conditions that we might have turned with confidence to any comprehensive tables of physical constants for information on this matter. It is, however, not an easy operation to measure the shearing modulus under a pressure of 10,000 kg. per sq. cm. Even Prof. Bridgman with all his long experience in experimentation under high pressures had to search for years for a suitable method. He says "With the development of the sliding contact potentiometer method of measuring small displacements, which I have already applied to the measurement of various small motions under pressure, it was evident that a means was at hand, and in this paper the application of such a method is described."

In order to get as large a displacement as possible under the applied pressure the helical spring was adopted as the advantageous form. It has the additional merit of compactness and permits the calculation of the shearing modulus from its geometrical data and its extension under load. A helical spring of hard drawn steel piano wire .025 cm. in diameter was used. It had an outside diameter of .75 cm. and length when stretched ready for use of 6.35 cm. with 48 turns. Such a spring hung vertically from a frame. Its lower end carried a vertical piece of manganin wire which in turn carried the weight that stretched the spring. An electric current traversed the manganin wire, the steel spring being one current lead and a long spiral of thin copper wire attached to the weight being the other. To the manganin wire was attached a potentiometer terminal. The same wire rested against a contact point joined to the potentiometer, by which the difference of potential between the terminal and the contact point was measured. Any change in length of the spring caused the contact point to rest on a different part of the manganin wire and caused the potential difference to assume a new value. From this the vertical motion of the spring was calculated. Since this motion under 6000 km./cm.^{2} is about .8 mm. only it was necessary to avoid errors due to friction of the contact point upon the wire. A slight tilting of the apparatus accomplished this by breaking the contact.

The steel spring, wire and weight were surrounded by a liquid and subjected to hydrostatic pressure. On account of the electrical current this liquid had to be an insulator. It should be relatively incompressible and its compressibility should be known. Kerosene met these conditions but had one marked disadvantage, under high pressure it became so viscous that the contact would not open by tilting at pressures above 6,000 kg./cm.^{2} As kerosene was used the pressure just mentioned was the highest that could be employed in the experiment. Under this pressure the steel spring shortened by .77 mm. This indicates an increase in the shearing modulus of about 1 per cent. This result was for 30° C. At 75° C. the kerosene was so much less viscous that readings could be made up to 12,000 km./cm.^{2} The results indicate a considerably smaller increase in the shearing modulus of steel under pressure at 75° C. than at 30° C.

An examination of the effect of pressure on the shearing modulus of glass was made by using an apparatus in which a stretched horizontal helix of glass and another of steel wire supported between them a horizontal piece of manganin wire against which the contact point rested. Upon the application of pressure the rigidity of the two substances was differently affected and the contact wire shifted in position. From the known value of the shearing modulus of steel under pressure that of glass was calculated. It was found that, in contrast to steel, the shearing modulus of glass grows less under increasing pressure, i.e., it is less rigid under high pressure whereas steel is more rigid. For six kinds of glass the pressure coefficients in kg. units ranged from -0.62 to -8.8×10^{-6}. That of steel is $+2.16 \times 10^{-6}$.

The effects of pressure on Young's Modulus and on Poisson's Ratio are calculated. For steel and all specimens of glass the pressure coefficient for Poisson's Ratio is positive, while the pressure coefficient of Young's Modulus is positive for steel and for two kinds of glass but negative for the remaining three specimens.

G. F. S.

Normal Atmospheric Dispersion as the Cause of the "Green Flash" at Sunset. LORD RAYLEIGH. (*Proc. Roy. Soc.*, A 801.) With a clear sky at the horizon as the upper limb of the setting sun is vanishing below the horizon it sometimes appears of a green color for the last 2 or 3 seconds. Scattered through the many volumes of the periodical Nature are many letters descriptive of this phenomenon. Formerly retinal fatigue was invoked as an explanation, but this is disproved by the occurrence of the effect at sunrise as well as at sunset. Atmospheric refraction with the accompanying dispersion is the cause of the green ray or flash. It is however in question whether the normal atomspheric dispersion at the horizon is adequate to account for the effect or must some unusual condition of the air occur as in the case of mirage. Lord Rayleigh sets himself the problem of determining by experiment which alternative must be accepted. Knowing the relative dispersion of the atmosphere for different lines of the solar spectrum he had a crown glass prism made with a dispersion of 22″ between the C and F lines. The angle of the prism was 45′. Sunlight was reflected into a long room through an opening in a shutter. In the opening was placed the prism with its refracting edge vertical. The beam was limited by a vertical straight edge at the thick end of the prism and by a second at the thin end. The refracted beam was caught on a screen 20 meters away. The bright patch was edged with red on the less refrangible side and with a less conspicuous blue-green on the opposite margin. "If we wish to have the blue-green color in the form of a 'flash,' a narrow strip of paper, say 1 mm. wide, may be substituted for the screen, its length being parallel to the colored fringe. Then, as the diurnal motion carries the colored edge of the luminous patch across the strip, the latter is momentarily seen illuminated with blue-green only. The duration of this flash should be about the same as that of the natural one, since both depend on the angular velocity of the sun. In accordance with this the experimental flash is found in fact to last about 3 seconds." If the eye be placed in the colored fringe, the green effect is lost because at the distance specified the separation from the red C line to the green F line is only 2.1 mm. and the pupil of the eye is not filled satisfactorily by the colored border. If the observer be on the deck of a ship and his horizon 10 km. away the distance between the C and F lines is 107 cm.

It is further shown that the effect of the diffraction pattern due to the horizon in confusing the spectrum is much less for long distances than for short ones.

The author is inclined to attribute the capricousness in the occurrence of the green flash to variation in atmospheric clearness, rather than to the presence of unusually large atmospheric dispersion near the horizon. If this actually occurs it should postpone the instant of sunset by as much as two minutes. The author hopes to get observations made in the tropics that will show whether such delay takes place when the green ray is seen.

In the laboratory experiment the color seen is not the distinct green seen in nature but a blue-green. When the light travels several km. before it reaches the eye of the observer the blue is scattered to a greater extent than the green, so that the fact that the last ray seen is green and not blue may be thus explained. At certain localities the green ray is frequently observed as in Granada in the West Indies and in California.

G. F. S.

Canadian Hydro-Electric Power Development during 1929. B. Cunningham. (*Nature*, May 31, 1930.) Two reports published by the Dominion Government at Ottawa, "Hydro-Electric Progress in Canada in 1929" and "Water Power Resources of Canada" furnish information both of the past and for the future. In 1929 378,000 H.P. of new electrical installations came into use bringing the total for the entire Dominion up to 5,727,162. The aggregate of all the present water power resources is about 43,000,000 H.P., but of this only about 13 per cent. is developed at the present time. The province of Quebec is credited with having the greatest available water power then follows Ontario with about half as much as Quebec. Prince Edward Island possesses a small amount of which 46 per cent. is used. Central Electric stations use about 84 per cent. of the horse power now installed. 11 per cent. goes to pulp and paper mills, 6 per cent. to other industries. In all Canada there are 312 hydro-electric plants. Of these the public controls 97 while the rest are owned by commercial enterprises. In the total number of horse power installed per thousand of population Yukon and Northwest Territory rank highest with 1063 and Saskatchewan lowest with .04. For the whole Dominion the figure is 572. The last named province appears to have but a single hydro-electric plant and that of only 35 horse power, though it has a million potential horse power.

G. F. S.

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